Central Clearing and Loss Allocation Rules

Dominic Cucic†
Danmarks Nationalbank

April 8, 2021

Abstract

I study the effects of central clearing in over-the-counter derivative markets in a simple model of derivative trading. When risk-sharing is limited by moral hazard problems facing protection sellers, central counterparties (CCPs) facilitate risk-sharing by mutualizing idiosyncratic counterparty risk and economizing on costly margin calls. When clearing members' defaults are correlated, CCPs optimally mutualize losses across its members. When CCPs face moral hazard problems too, absorbing losses with CCP capital is essential to provide risk-management incentives to the CCP. A clearing fee can compensate CCPs for raising costly capital but may be prohibitively costly and warrant a return to bilateral trading.

JEL classification: G22, G28, D82

Keywords: Moral hazard; Central clearing; Counterparty risk; Margins; Capital

†Danmarks Nationalbank, Langelinie Alle 47, 2100 Copenhagen. Email: dominic.cucic@gmail.com. Web: www.sites.google.com/view/dominiccucic/. I am grateful to Javier Suarez for his excellent guidance and continuous support. I also thank Gideon Saar (the Editor), the anonymous referee as well as Rafael Repullo, David Martinez-Miera, and Andrés Almazán for their helpful comments. I acknowledge the financial support received from CEMFI during this project. Any remaining errors are my own. Views expressed herein are those of the author and do not necessarily reflect those of Danmarks Nationalbank.

1 Introduction

The global financial crisis (GFC) of 2007-08 highlighted the threat that markets for over-the-counter (OTC) derivatives may pose to financial stability. As a consequence, regulators in the US and in Europe required that all standardized OTC derivatives contracts be cleared through central counterparties (CCPs). Effectively, the reform replaced the complex bilateral trading structure with a centralized network centered around CCPs. CCPs insert themselves between the participants of a derivatives transaction and guarantee payments that would be subject to counterparty default risk in a bilateral transaction. Due to these regulatory reforms, the importance of central clearing has risen markedly over the last two decades: for example, by the first half of 2018 over 75% of the notional value of interest rate swaps (IRS) and credit default swaps (CDS) - the two largest categories of OTC derivatives affected by clearing mandates - was centrally cleared (King et al., 2020).

Because of the sheer size of OTC derivative markets and the importance of central clearing in the new market structure, it is crucial to understand the role of CCPs in promoting (or endangering) financial stability. This paper contributes to this discussion by developing a tractable theoretical framework of derivative trading to address open questions related to the optimal design and governance of CCPs. In particular, it studies if, and how, CCPs promote risk-sharing when aggregate shocks induce correlated defaults by clearing members. In addition, the paper analyzes the effect of moral hazard problems at the CCP level on the provision of insurance and how central clearing arrangements can be structured to alleviate such problems. The analysis produces clear implications for the design of CCPs' loss allocation rules, that is how CCPs use various available resources to absorb clearing members' defaults. The results also contribute to the ongoing debate regarding the role of CCP capital (FSB, 2015).

The analysis is based on a stylized model of derivative trading in which the CCP, as well as protection sellers, face moral hazard problems. The model features risk-averse protection buyers, e.g. commercial banks, who want to insure themselves against a common aggregate risk exposure. In line with the bilateral trading structure before regulatory reforms, protection buyers can directly contract risk-neutral protection sellers, e.g. investment banks or insurance companies, who are able to invest in risky assets. When investing in risky assets, protection sellers can carefully select safer assets or shirk and enjoy private benefits derived from their gamble on more risky assets. Importantly, shirking protection sellers increase counterparty risk for protection buyers. Since it is difficult to observe and verify how financial institutions (protection sellers) judge the riskiness of different assets, protection sellers face a moral hazard problem. This is one of the key frictions in the analysis.

When the moral hazard problem is severe, protection buyers can require margin deposits from protection sellers. Margin calls require sellers to invest a fraction of their initial cash holdings into safe assets, thereby insulating them from sellers' moral hazard problem and thus improving risk-prevention incentives. However, investing into margin accounts is costly as the funds forgo the excess return of sellers' risky asset.

Since protection buyers are infinitely risk averse, the derivative contract must provide them with full insurance. However, in a bilateral contract protection buyers are always exposed to some counterparty risk because even prudently managed risky assets may fail. Therefore, the only way to guarantee the promised payments to protection buyers is to have them fully collateralized, that is all of the promised payments must be backed up with cash in safe margin accounts. While protection buyers are always exposed to their counterparty's default risk in a bilateral trade, the CCP finances insurance payouts by pooling resources from *all* protection sellers. Consistent with existing literature, a CCP may thus improve upon the bilateral trading benchmark by mutualizing idiosyncratic counterparty risk.[1] Following an optimal contracting approach clarifies how margin requirements and central clearing interact: while margin calls provide incentives to reduce counterparty risk, central clearing mutualizes idiosyncratic counterparty risk and thereby reduces the need for costly margin calls.

[1] See for example Biais, Heider and Hoerova (2015).

The focus of the analysis is on the effectiveness of central clearing arrangements in the face of correlated defaults by protection sellers. Considering correlation in the outcomes of clearing members is especially relevant when CCPs specialize their operations regionally or in terms of the types of cleared derivatives. The results show that efficient risk-sharing requires the CCP to mutualize losses across clearing members: non-defaulting protection sellers' payment to the CCP is higher when more sellers default, which allows the CCP to insure protection buyers against the aggregate shock driving correlated defaults by sellers.

This feature of the model provides a theoretical rational for the design of CCPs' "loss allocation rules" (so-called default waterfalls), which is highly relevant since there are no regulations regarding the design of these waterfalls and empirically there is heterogeneity in their design across CCPs (Carter and Garner, 2015). In practice, many CCPs require not only margin deposits from clearing members but also a "default fund" contribution, which the CCP may draw upon when it faces large losses from clearing members' defaults. In line with the model's predictions, using default fund contributions of *non-defaulting* members allows the CCP to mutualize theses losses.

However, whether central clearing facilitates the provision of insurance in the face of correlated defaults depends crucially on the realized default rates. Higher default rates require the CCP to mutualize larger losses, which it achieves by collecting higher payments from non-defaulting sellers. As sellers are protected by limited liability, non-defaulting sellers' cannot pay the CCP more than the combined return of their risky asset and margin accounts. Consequently, the extent of risk-sharing achieved in central clearing arrangements may be limited when default rates among clearing members are high. The results show that moderately high default rates may be sufficient to eliminate the benefits of central clearing.

Finally, I study how the design of loss allocation rules and central clearing arrangements is affected by moral hazard problems at the CCP level. Therefore, I extend the model so that the CCP can monitor protection sellers to rein in their private benefits.

However, monitoring is costly since it requires personnel that is trained in evaluating the complex financial conditions of clearing members. Since monitoring efforts by the CCP are impossible to observe for outsiders this creates a moral hazard problem for the CCP. This is the second key friction in the model.

I show how to design a compensation scheme for the CCP's owners, in the form of clearing fees and CCP capital, that minimizes counterparty risk. Moral hazard problems at the CCP level create a dual role for CCP capital similar to the one of margin deposits: first, capital provides (costly) resources that can absorb large losses from defaulting clearing members. Second, capital incentivizes diligent monitoring efforts by the CCP. Since CCP monitoring alleviates the incentive problem of protection sellers, monitoring makes the provision of risk-prevention efforts to sellers cheaper. As the CCP has to absorb smaller losses when protection sellers manage assets prudently, CCP owners are inclined to induce monitoring when the capital they provided to the CCP is at risk.

However, the CCP must be compensated for raising costly capital. I show that a clearing fee can be used to compensate the CCP. Effectively, a clearing fee allows the CCP to pass on the cost of providing monitoring incentives to clearing members. The results highlight that even mild moral hazard problems at the CCP warrant sizeable clearing fees. Clearing fees may in fact be so large that they make central clearing unappealing to protection buyers.

The analysis is based on a modified version of the model of Biais, Heider and Hoerova (2015) (henceforth "BHH"). While BHH focus on the interaction between central clearing and margins, this paper focuses on the design of CCPs' loss allocation rules in the face of correlated defaults among clearing members and moral hazard problems at the CCP level. Adopting the BHH framework to analyze these questions requires extending their analysis along some dimensions and, to keep tractability, simplify it along others. Specifically, I consider the case when prudent risk-management by protection sellers does not eliminate default risk. Additionally, I highlight that CCP owners must be compensated to ensure functioning of central clearing and diligent monitoring efforts by the CCP. The analysis is simplified by abstracting from public signals about the aggregate state of the economy (which was essential for the discussion of signal contingent contracts by BHH) and by assuming that protection buyers are *infinitely* risk-averse, which allows to focus on the implementation of full insurance contracts.

Several aspects of the analysis resemble features of the model of Holmström and Tirole (1997). First, I adopt the modeling of moral hazard where protection sellers choose between effort and shirking. Second, I borrow the term "pledgeable income" to refer to the future income generated by risky assets that can be promised to be paid without undermining risk-prevention efforts. In addition, the analysis by Holmström and Tirole (1997) also features two layers of moral hazard problems, since both firms as well as banks must be incentivized to behave in the interest of their financiers. In addition, the treatment of the incentive problem at the CCP is very similar to the problem of the monitoring intermediary. Therefore, I extend one of their key results to central counterparties: a monitoring CCP must satisfy an incentive-based capital adequacy ratio.

Although the literature on insurance usually focuses on moral hazard problems for protection buyers, the assumption of moral hazard facing protection sellers is not unique. Thompson (2010) studies protection sellers who face moral hazard. However, in his setting moral hazard facilitates the provision of insurance by reducing problems arising due to adverse selection. Despite assuming moral hazard at the same side of insurance contracts, the present model is quite different since I do not study adverse selection problems and moral hazard in this setting limits the feasible amount of insurance.

My work is connected to the asset pricing literature that studies how central clearing affects systemic and counterparty risk. Amini, Cont and Minca (2016) study the systemic risk implications of CCPs being unable to fully absorb member defaults and derive a CCP fee and guarantee fund policy structure that reduces systemic risk. Duffie and Zhu (2011) conclude that a CCP dedicated to a specific class of derivative contracts will on average lead to a higher exposure to counterparty defaults. This result arises because a CCP decreases the opportunities for bilateral netting between two contracting parties that trade in different derivative classes. Since a CCP typically clears only few specific products, trading parties can no longer net their positions in centrally cleared products with positions in uncleared derivatives. This paper does not analyze the effect of central clearing on netting efficiency and I instead find that CCPs *reduce* counterparty risk. Garratt and Zimmerman (2015) reach similar conclusions in their analysis of central clearing. However, they show that highly risk averse agents can benefit from central clearing since it reduces the variance of netting exposures. In line with this result, my model features infinitely risk-averse protection buyers who benefit from CCP arrangements, unless the CCP faces moral hazard.

In their study of frictions in derivative markets Acharya and Bisin (2014) take a different approach from the model in this paper: they focus on externalities that arise when several protection buyers contract with the same protection buyer. They highlight that pricing schedules under central clearing can be designed to to internalize these externalities. In contrast, bilateral trading in my model takes place between a single buyer and seller of insurance and creates no externalities. Instead, I focus on moral hazard problems which can be alleviated by setting prices and quantities in an incentive compatible way.

In recent years a growing literature has analyzed various other aspects of central clearing arrangements. On the theoretical side, Koeppl, Monnet and Temzelides (2012) find that when derivative clearing is costly, it may be efficient to subsidize bilateral trading arrangements by charging higher clearing fees at CCPs. Fontaine, Perez-Saiz and Slive (2014) examine entry restrictions for clearing members. Menkveld (2017) focuses on CCPs' risk management tools and proposes a new exposure measure based on tail risk in trader portfolios. Faruqui, Huang and Takáts (2018) consider how feedback loops between CCPs and banks can be destabilizing. Kuong and Maurin (2020) study how optimal clearing arrangements and CCP ownership are affected by market size and cost of collateral. On the empirical front, Duffie, Scheicher and Vuillemey (2015) estimate the impact of central clearing and margin regulations on the demand for collateral. Vuillemey (2020) provides a case study of the introduction of the first CCP in history. Bignon and Vuillemey (2020) analyze how risk management failures contributed to the historic collapse of a CCP.

The remainder of the paper is structured as follows. The model is presented in Section 2, which also analyzes the bilateral trading benchmark. I then analyze central clearing and the benefits of CCP arrangements in Section 3. Section **??** highlights that these benefits are unaffected by the introduction of aggregate shocks inducing correlation in the outcomes of protection sellers' investments. Section 4 stresses that moral hazard at the level of the CCP can severely limit the provision of insurance to protection buyers. Section 5 discusses several implications for the design of central clearing arrangements before Section 6 concludes.

2 Model and bilateral trading benchmark

Consider an economy with three dates indexed by $t \in \{0, 1, 2\}$. There are three classes of agents: buyers and sellers of protection and a central counterparty (CCP). Protection buyers and sellers consume only at the final date and do not discount their future consumption. The CCP can write derivative contracts with protection buyers and sellers, but does not consume itself. At $t = 0$, the relevant agents design the derivative contract whereas investment decisions are made at $t = 1$. Final payoffs are realized at $t = 2$.

Protection buyers. There is a continuum of identical and infinitely risk-averse protection buyers with measure one. At $t = 0$ each protection buyer is endowed with one unit of a long-term, non-tradeable, risky asset with payoff at $t = 2$ given by

$$\tilde{\theta} = \begin{cases} \bar{\theta} & \text{with prob.} \quad \pi \\ \underline{\theta} & \text{with prob.} \quad 1 - \pi \end{cases},$$

with $\bar{\theta} > \underline{\theta}$. All protection buyers are subject to the same aggregate risk driving the realization of $\tilde{\theta}$ and thus cannot hedge against it by trading with other protection buyers. They can, however, insure against this aggregate risk by purchasing insurance, in the form of derivative contracts, from protection sellers. An application of this setting may be protection buyers purchasing long-term put options to hedge against macro risk factors. Importantly, I assume that each buyer deals with at most one protection seller (or one CCP). Hence I abstract from the possibility that a given protection buyer directly contracts with a continuum of protection sellers.

The insurance contract specifies an insurance payout denoted by Y, which buyers receive in the bad aggregate state $\underline{\theta}$. In addition, the contract specifies an insurance payment denoted by X, which buyers pay to protection sellers in the good aggregate state $\bar{\theta}$. Because of their infinite risk-aversion, protection buyers only accept contracts offering full insurance. In other words, the contract must equalize their consumption levels across all states of nature. Therefore the payments specified by the derivative contract must satisfy the full insurance condition $\bar{\theta} - X = \underline{\theta} + Y$, which can be written as

$$X + Y = \bar{\theta} - \underline{\theta} \equiv \triangle\theta. \tag{1}$$

Under full insurance, protection buyers' utility is independent of the final state: $u^B = u(\underline{\theta} + Y) = u\left(\bar{\theta} - X\right)$. When deriving the optimal contracts in subsequent sections, I design contracts to maximize protection buyers utility. Clearly, this is equivalent to maximizing the insurance payment buyers receive in the bad aggregate state, Y (or minimizes the insurance payment they have to make in the good state, X). Hence the optimal contracting problem reduces to finding the maximum insurance payout Y compatible with a number of constraints, which I discuss in the following sections.

Protection sellers. There is a measure one continuum of risk-neutral protection sellers indexed by j who are endowed with one unit of a divisible, safe asset such as Treasury bills and protected by limited liability. At $t = 1$ protection sellers invest their endowment across the safe asset and a divisible, risky asset with a $t = 2$ payoff denoted by $\tilde{R}_j$. The risky asset pays R per unit with probability $p_j \in (0, 1)$ and fails to pay anything otherwise. Protection sellers have unique skills in managing the risk associated with this asset, allowing them to influence the success probability p_j. To simplify the role of risk-management I assume that each protection seller j can exert effort ($e_j = 1$) at $t = 1$ to reduce the riskiness of her asset. In other words, risk-prevention efforts by a seller improves the return of her assets in the sense of first order stochastic dominance:

$$p_j = \begin{cases} p_h & \text{if } e_j = 1 \\ p_l & \text{if } e_j = 0 \end{cases}$$

where $p_l < p_h < 1$. This assumption implies that even a prudently managed risky asset may fail. Conditionally on the choice of effort, the realization of the risky assets' returns are independently distributed across protection sellers.

Because the risk-management effort of protection sellers is based on their unique skills, it is difficult to observe and monitor by outside parties. In addition, exerting effort is costly for protection sellers because they forgo a private benefit B per unit of assets under management at $t = 1$. A protection seller's private benefit B_j is thus given by $B > 0$ if she shirks ($e_j = 0$) and by 0 otherwise ($e_j = 1$). I assume that undertaking effort is efficient and delivers higher expected returns than investing in the safe asset:

$$p_h R > \max \{p_l R + B, 1\}. \tag{A1}$$

This assumption implies that the incentive problem related to risky assets is not too large. In particular, the return of the risky asset which can be pledged to protection buyers, the "pledgeable return" $\mathcal{P}$ defined by Holmström and Tirole (1997), is positive:

$$\mathcal{P} \equiv R - \frac{B}{p_h - p_l} > 0.$$

Margin calls. Insurance contracts may require margin deposits, which are assumed to be contractible and enforceable. The party subject to a margin call is required to deposit a given amount of the safe asset into a separate account at $t = 1$. Importantly, once the safe asset is invested into a margin account it cannot be accessed before the contract expires at $t = 2$. Because protection buyers have no ability to manage risky assets, margin deposits must be satisfied with safe assets. Therefore assets in the margin account are effectively protected from moral hazard but they forgo the excess return of the (prudently managed) risky asset, $p_h R - 1$. Hence the opportunity cost of margins is directly related to the risk of protection sellers' assets and therefore help in providing risk-prevention efforts. I denote the fraction of protection sellers' assets in margin accounts by α, so contracts must satisfy the feasibility constraint on margins

$$\alpha \in [0, 1]. \tag{2}$$

Since protection sellers are constrained by limited liability, their payments in the bad aggregate state $\underline{\theta}$ must satisfy the limited liability condition

$$Y \leq \alpha, \tag{3}$$

which ensures that the payments Y can be financed with margin deposits if the risky asset returns $R_j = 0$. Limited liability in combination with the unobservable effort choice by protection sellers generates moral hazard.

2.1 Bilateral trading

This section studies a bilateral contract between a single buyer and seller of protection, which serves as the benchmark case against which I will compare various centrally cleared arrangements. The bilateral contract specifies a payment X from the buyers to the seller in the good aggregate state $\bar{\theta}$ and a payment Y from the seller to the buyer in the bad aggregate state $\underline{\theta}$. Moreover, the contract specifies margin calls α.

Table 1 summarizes the payoffs for a single pair of protection buyers and sellers $\{B, S\}$ for any bilateral contract depending on the realization of the risky assets $\tilde{\theta}$ and $\tilde{R}_j$.

Forming expectations about protection sellers' payoff in each state of the world de-

Payoffs $\{B,S\}$	$\tilde{R}_j = R$	$\tilde{R}_j = 0$
$\tilde{\theta} = \bar{\theta}$	$\{\bar{\theta} - X, \quad \alpha + (1-\alpha)R + X\}$	$\{\bar{\theta} - X, \quad \alpha + X\}$
$\tilde{\theta} = \underline{\theta}$	$\{\underline{\theta} + Y, \quad \alpha + (1-\alpha)R - Y\}$	$\{\underline{\theta} + Y, \quad max\{\alpha - Y, 0\}\}$

Table 1: Payoffs of protection buyers and sellers under a bilateral contract

This table summarizes the final payoffs to protection buyers (B) and sellers (S) under a bilateral contract depending on the realizations of risky asset returns $\{\tilde{\theta}, \tilde{R}_j\}$.

picted in Table 1 shows that their expected payoff for a given choice of p_j is

$$u^S(p_j) = \pi\left[\alpha + X + p_j(1-\alpha)R\right] + (1-\pi)\left[\alpha - Y + p_j(1-\alpha)R\right] + (1-\alpha)B_j. \quad (4)$$

The good aggregate state materializes with probability π, in which case sellers receive their margin deposit α, the insurance payment X from protection buyers and the expected payoff p_jR of the $(1-\alpha)$ units of the risky asset. The adverse aggregate state materializes with probability $1-\pi$, in which case sellers receive their margin deposit net of the payment to protection buyers $\alpha - Y$ and the expected payoff of their risky asset. Regardless of the aggregate state, sellers receive private benefits B_j per unit of their risky asset.

For ease of exposition I now focus on the optimal contract that induces prudent risk management by sellers. Proposition 1 summarizes the optimal bilateral contract.

Proposition 1 *(Bilateral Contract): The optimal bilateral contract providing risk-prevention incentives features full collateralization with* $\alpha = \hat{Y}^* = \min\left\{\frac{\pi\triangle\theta}{p_hR}, 1\right\}$.

Proof: I derive the optimal bilateral contract by maximizing the insurance payout to protection buyers Y while guaranteeing protection sellers' participation and risk-prevention incentives. Moreover, the contract is subject to the full insurance condition (1), the feasibility constraint on margins (2) and the limited liability constraint (3).

Since each protection buyer seeks full insurance against her risk $\tilde{\theta}$ and always faces counterparty risk ($p_h < 1$), the insurance payout in the bilateral contract must be fully collateralized, that is $Y = \alpha$. Backing up the full insurance payout with safe margin deposits is the only way to guarantee the payment of Y when a given seller's risky asset returns $\tilde{R}_j = 0$. Since margin deposits are costly, the optimal contract will not call for higher margins than necessary to guarantee the payment of Y.

First, it provides useful to simplify sellers' expected payoff in (4) by focusing on the fully collateralized, effort inducing contract $(Y = \alpha, p_j = p_h, B_j = 0)$, and utilizing the full insurance condition (1), which implies $X = \triangle\theta - Y$:

$$u^S(p_h) = \pi\triangle\theta + (1 - Y)p_h R. \tag{5}$$

In order for protection sellers to agree to the optimal contract, participation in the effort inducing contract must at least compensate them for their stand-alone expected profit from fully investing in the risky asset (which yields higher expected returns than investing into safe assets by (A1)). Therefore, sellers' participation constraint is

$$\pi\triangle\theta + (1 - Y)p_h R \geq p_h R. \tag{6}$$

Protection sellers' incentive constraint requires that exerting effort compensates for the loss of private benefits

$$\pi\triangle\theta + (1 - Y)p_h R \geq \pi\triangle\theta + (1 - Y)[p_l R + B],$$

which is equivalent to $\mathcal{P} \geq 0$ and satisfied by assumption (A1). When sellers' pledgeable income is positive, a fully collateralized contract leaves no residual incentive problem. Determining the optimal effort inducing bilateral contract then reduces to finding the largest insurance payout Y still guaranteeing sellers' participation. Solving their binding participation constraint (6) for Y yields $\hat{Y} = \pi\triangle\theta/(p_h R)$.

Using the full collateralization result makes it trivial to show that the feasibility condition on margin calls and the limited liability condition of the bilateral contract coincide. These constraints require $\alpha = \hat{Y} \leq 1$, which implies that the optimal insurance payout in the bilateral contract is

$$\hat{Y}^* = \min\left\{\frac{\pi\triangle\theta}{p_h R}, 1\right\}. \quad \blacksquare$$

The insurance payout under the effort-inducing bilateral contract described in Proposition 1 is intuitive: it is increasing in the riskiness of buyers' asset $\triangle\theta$, which increases buyers' insurance need, and decreasing in the expected return of sellers' risky asset, as this improves sellers' outside option and makes it harder to guarantee their participation in the contract. Moreover, the insurance payout can not increase above 1, the maximum amount that sellers' can deposit in the margin account.

Going forward I additionally assume that the insurance payment $\hat{Y}$ is feasible, so that protection sellers who exert effort can fully insure buyers:

$$p_h R \geq \pi\triangle\theta. \tag{A2}$$

Buyers payment' X to sellers in the good aggregate state is determined by replacing $\hat{Y}$ in the full insurance condition (1), which yields $\hat{X} = \triangle\theta[1-(\pi/(p_h R))]$. The payment sellers' receive compensates for providing insurance and is naturally increasing in both the size of the risk they insure $\triangle\theta$ and their outside option of investing fully into their own risky asset $\tilde{R}$.

Lastly, the optimal bilateral contract is not actuarially fair since the expected transfer to protection sellers,

$$\pi\hat{X} - (1-\pi)\,\hat{Y} = (p_h R - 1)\,\hat{\alpha},$$

is positive. Compensating protection sellers for the efficiency loss induced by costly margin calls requires paying them a premium that is increasing in the opportunity cost of margins $(p_h R - 1)$ and the size of the margin call $\hat{\alpha}$.

3 Central clearing

In this section I explore the effects of contracting with a central counterparty. The main question is whether central clearing can improve upon the insurance provided by bilateral trades, that is, whether the optimal contract with central clearing can deliver an insurance payout that is higher than $\hat{Y}$. Clearly the CCP could always replicate the bilateral contract but it may improve upon this contract by mutualizing idiosyncratic counterparty risk. Following Biais, Heider and Hoerova (2015), the CCP can pool resources from non-defaulting protection sellers and use it to make transfers to protection buyers, thus providing insurance against counterparty risk.

To study if central clearing is only beneficial in the face of purely idiosyncratic counterparty risk, I introduce additional uncertainty. In particular, I assume that a CCP-specific shock hits all sellers contracting with a given CCP at time $t = 2$ (independently from the shock to buyers' risky asset $\tilde{\theta}$) before asset values are realized.[2] The shock affects the riskiness of protection sellers' risky asset $\tilde{R}$:

$$p_j = (1 - \epsilon)\, p_{jg} + \epsilon p_{jb} \quad \text{for} \quad p_{jg} > p_j > p_{jb}$$

where $s = \{g, b\}$ indicates a CCP-specific state and $j = \{h, l\}$ identifies the risk-management choice of protection sellers. I assume that the CCP-specific shock does not change the effect of sellers' risk-prevention efforts on their risky asset's success probability:

$$p_{hg} - p_{lg} = p_{hb} - p_{lb} \equiv \triangle p \tag{A3}$$

The CCP-specific shock effectively introduces correlation into protection sellers' outcomes, thereby potentially limiting the benefits derived from central clearing.

CCP contracts. In a centrally cleared market, all previously bilateral agreements

[2] The CCP-specific shock can be thought of as a result from specialization at CCPs. With specialization, e.g. regionally or in terms of the types of cleared derivatives, clearing members of a given CCP are likely to be subject to the same shocks.

between buyers and sellers are replaced with contracts between these buyers and sellers and the risk-neutral CCP in a process called "novation". In this process, the CCP sets up separate contracts at $t = 0$, specifying transfers to and from protection buyers $\{X, Y\}$ as well as sellers $\{x, y_g, y_b\}$. In the good aggregate state $\overline{\theta}$ protection buyers pay X to the CCP while the CCP pays out x to sellers. In the bad aggregate state $\underline{\theta}$ buyers receive Y from the CCP, which the CCP finances from two sources: margin calls α from defaulting sellers, and a payment $y_s, s = \{g, b\}$ from non-defaulting sellers to the CCP. Non-defaulting protection sellers' payment depends on the realization of the CCP-specific shock s since it affects the fraction of defaulting sellers and thereby the resources available to the CCP for distribution among buyers. This leads to a modified limited liability constraint for protection sellers, which requires that non-defaulting sellers' payments to the CCP in the bad aggregate state $\underline{\theta}$ do not exceed their available resources:

$$\max\{y_g, y_b\} \leq (1 - \alpha)R + \alpha. \tag{7}$$

All transfers are made once the realizations of $\left\{\tilde{\theta}, \tilde{R}_j\right\}$ are known at $t = 2$. Since the transfers are contingent on final asset values, they can be interpreted as transfers specified by derivative contracts.

In a centrally cleared market the payment to a protection buyer Y depends on all sellers' asset returns as opposed to those of a single seller. Naturally, the transfers at $t = 2$ are subject to a resource constraint: payments to protection buyers Y (sellers X) in the bad aggregate state $\underline{\theta}$ (good agg. state $\overline{\theta}$) cannot exceed the resources provided by all protection sellers (buyers). Assuming that the CCP aggregates a mass of relationships of measure 1, this leads to the aggregate resource constraints

$$X = x \text{ and } Y = p_{hg}y_g + (1 - p_{hg})\,\alpha = p_{hb}y_b + (1 - p_{hb})\,\alpha, \tag{8}$$

where I have focused on the contract implementing high effort by protection sellers. Since buyers require full insurance, the aggregate resource constraint in the bad state $\underline{\theta}$ ensures that the total amount of funds collected from sellers by the CCP is not affected by the CCP-specific shock. Since the CCP collects resources from all protection sellers for distribution, the process of novation allows to mutualize idiosyncratic counterparty risk. Note that the bilateral contract can be interpreted as a CCP contract with aggregate constraints $X = x$ and $Y = y_s$.

The optimal CCP contract. I now proceed to determine the optimal centrally cleared contract by maximizing the insurance payout Y to protection buyers while guaranteeing sellers' participation and risk-prevention efforts. First, consider protection sellers' expected payoff when exerting effort under central clearing:

$$u^S(p_h) = (1-\alpha)\,p_h R + [\pi + (1-\pi)p_h]\,\alpha + \pi x - (1-\pi)\,[(1-\epsilon)p_{hg}y_g + \epsilon p_{hb}y_b]\,. \quad (9)$$

Despite the CCP-specific shock the expected payoff of prudent protection sellers' $(1-\alpha)$ units of the risky asset is still given by $p_h R$. In a centrally cleared market, prudent protection sellers keep their margin deposits unless buyers' risky asset and the CCP are both hit with an adverse shock, that is the economy is in state $(\underline{\theta}, b)$, which occurs with probability $(1-\pi)(1-p_h)$. Sellers' receive x from the CCP in the good aggregate state, while their payment in the bad aggregate state varies with the CCP-shock: they face a favorable CCP-shock $(s = g)$ with probability $1-\epsilon$, in which case their payment to the CCP is y_g, while a unfavorable CCP-shock realizes with probability ϵ, in which case their payment to the CCP is y_b.

Guaranteeing protection sellers' participation in the centrally cleared arrangement requires that it delivers a higher expected payoff than fully investing in their risky asset $\tilde{R}$. Sellers' participation constraint is thus $u^S(p_h) \geq p_h R$, which after replacing from (9) yields

$$\pi X - (1-\pi)\,[(1-\epsilon)\,p_{hg}y_g + \epsilon p_{hb}y_b] \geq [p_h R - \pi - (1-\pi)\,p_h]\,\alpha.$$

The expected transfers from the CCP to a protection seller (left-hand-side) must be high enough to compensate her for the opportunity cost of the expected use of margins (right-hand-side). Thus, if margins are used, the contract is not actuarially fair.

It will provide useful to write sellers' participation constraint in terms of only two features of the optimal contract, the optimal margin call α and non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, g)$, y_g. Consequently, I now proceed to replace X and y_b in the participation constraint. First, combining the CCP's aggregate resource constraints (8) and the full insurance condition (1) yields

$$X = x = \triangle\theta - p_{hg}y_g - (1 - p_{hg})\,\alpha. \tag{10}$$

Since the CCP's resource constraints require that the aggregate payment to the CCP in state $\underline{\theta}$ is unaffected by the realization of the CCP-specific shock s, combining the resource constraints for the insurance payout Y across states s yields

$$y_b = \frac{1}{p_{hb}}\left[p_{hg}y_g - (p_{hg} - p_{hb})\,\alpha\right]. \tag{11}$$

Replacing X and y_b from above in protection sellers' participation constraint yields

$$y_g \leq \frac{\lambda}{1-\pi} - \left[\frac{p_h R}{p_{hg}} - 1\right]\alpha, \tag{12}$$

for

$$\lambda \equiv \frac{\triangle\theta\pi\,(1-\pi)}{p_{hg}}. \tag{13}$$

Since assumption (A1) implies $p_h R > 1 \geq p_{hg}$, the participation constraint suggests that margin calls tighten sellers' participation constraint and reduce the insurance payment from protection sellers' whose risky asset do not fail.

However, margin calls may be necessary to provide ex-ante risk-prevention incentives to sellers. Since undertaking effort is efficient by assumption (A1), the contract is designed such that it is in the interest of protection sellers to manage their risky asset prudently. The resulting incentive compatibility constraint for protection sellers requires that exerting risk-prevention efforts compensates them for the foregone private benefits, that is $u^S(p_h) \geq u^S(p_l)$, where slacking sellers' expected payoff under central clearing is

$$u^S(p_l) = p_l(1-\alpha)R + [\pi + (1-\pi)p_l]\alpha + \pi x - (1-\pi)[(1-\epsilon)p_{lg}y_g + \epsilon p_{lb}y_b] + (1-\alpha)B.$$

Replacing this, together with $u^S(p_h)$ from equation (9), and re-arranging, yields the incentive compatibility constraint

$$(1-\epsilon)y_g + \epsilon y_b \leq (1-\alpha)\frac{\mathcal{P}}{(1-\pi)} + \alpha.$$

The left-hand side is what protection sellers expect to pay to the CCP when buyers' risky asset return is low, that is when $\tilde{\theta} = \underline{\theta}$. The right-hand side is the sum of the pledgeable return on i) the assets deposited on the margin account and on ii) those left under protection sellers' management. The pledgeable return on assets deposited on the margin account is equal to their physical return since they are "ring-fenced" from moral hazard in risk-management.

To write this constraint in a similar fashion to the participation constraint (12) as a function of only y_g and α, I utilize equation (11) to replace y_b in protection sellers' incentive compatibility constraint, which yields

$$y_g \leq \frac{q(\epsilon)\mathcal{P}}{1-\pi} + \left(1 - \frac{q(\epsilon)\mathcal{P}}{1-\pi}\right)\alpha, \tag{14}$$

where

$$q(\epsilon) \equiv \frac{p_{hb}}{[p_{hb} + \epsilon(p_{hg} - p_{hb})]}.$$

is a factor that is strictly decreasing in the probability of a bad CCP-specific shock ϵ and satisfies $q(\epsilon) \in [p_{hb}/p_{hg}, 1]$. Equation (14) shows that margin calls α may allow to relax protection sellers' incentive constraint, that is, they can facilitate the provision of prudent risk-management incentives. However, this is not necessarily the case and whether margin calls help or hurt in providing incentives to sellers depends on whether $\mathcal{P} \leq (1-\pi)/q(\epsilon)$ or $\mathcal{P} > (1-\pi)/q(\epsilon)$ holds, respectively.

Figure 1 shows the regions of feasible contracts determined by the participation and incentive constraints, (12) and (14) respectively. Combinations of margin calls α and non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, g)$, y_g, that are compatible with the constraints are located below the respective constraints in the graph. Figure 1 additionally highlights how the sets of feasible contracts changes with the severity of protection sellers' moral hazard problem. As private benefits increase, and pledgeable income declines, the set of feasible contracts shrinks, optimal margins grow and the optimal insurance payment by non-defaulting sellers in aggregate state $(\underline{\theta}, g)$, y_g, declines. Consequently, protection buyers are worse off when sellers' moral hazard problem is severe since the optimal contract has to offer sellers higher rents in order to induce prudent risk-management efforts.

Figure 1 illustrates that there are three possible configurations of the optimal centrally cleared contract. When sellers' moral hazard problem is not severe, that is when pledgeable income is high, the optimal contract is determined by sellers' binding participation constraint (Point A) and their incentive constraint is slack. When pledgeable income is intermediate, the optimal contract is determined by sellers' binding incentive constraint (Point B) and their participation constraint is slack. Finally, when pledgeable income is low the optimal contract is determined by the intersection of the participation and incentive-constraint.

Solving for the optimal centrally cleared contract now reduces to maximizing the insurance payout to buyers Y by determining the optimal combination of non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, g)$, y_g, and margin call α. The maximization problem is subject to protection sellers' participation and incentive-constraints, (12) and (14) respectively, as well as their limited liability condition (7) and the feasibility constraint on margins (2). After determining the tuple (y_g, α), I show that it is straightforward to recover the remaining features of the optimal contract (y_b, x, Y, X).

To determine the optimal tuple (y_g, α) it is necessary to establish two facts about protection sellers' incentive- and participation constraints: how their intercepts compare

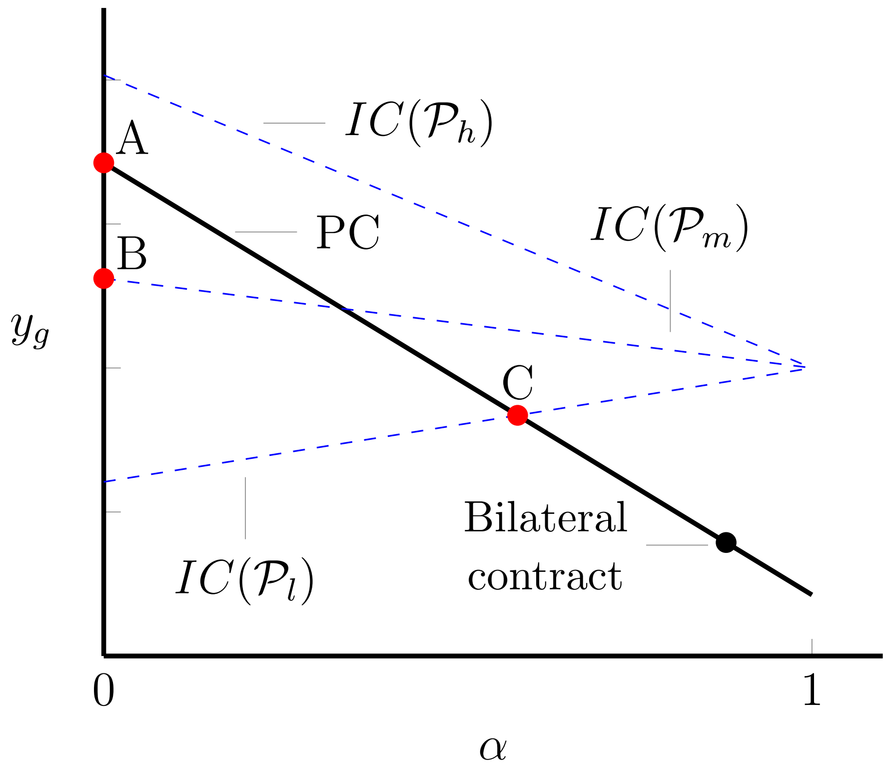

Figure 1: How optimal CCP contracts vary with private benefits

This Figure illustrates how protection sellers' participation and incentive compatibility constraints, PC and IC respectively, restrict non-defaulting sellers' payment to the CCP y_g in state $\underline{\theta}$ as functions of margin calls α. It additionally shows how the incentive constraint varies with sellers' pledgeable income by illustrating the IC for three possible values of $\mathcal{P}$ with $\mathcal{P}_l < \mathcal{P}_m < \mathcal{P}_h$. Participation and incentive compatible contracts are located below the respective constraints. Red dots mark the optimal contracts for each of the three plotted incentive constraints, while the black dot marks the optimal bilateral contract.

and whether the incentive constraint is upward or downward-sloping. As Figure 1 depicts, for optimal margins to be positive the incentive constraint must be upward-sloping ($\mathcal{P} \leq (1-\pi)/q(\epsilon)$) so that margins provide ex-ante risk-prevention incentives for protection sellers. In addition, for optimal margins to be positive, the intercept of the incentive constraint (14) must be below the participation constraint's (12) intercept, which happens if $\mathcal{P} \leq \lambda/q(\epsilon)$. In summary, optimal margins are positive if and only if sellers' pledgeable income is low:

$$\mathcal{P} \leq \min\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\} \tag{15}$$

If this condition is violated, the optimal contract under central clearing does not require margin calls. What remains to be shown then is whether non-defaulting sellers' payment y_g to the CCP in aggregate state $(\underline{\theta}, g)$ is determined by sellers' participation, incentive, or limited liability-constraint. Figure 1 illustrates that an increase in sellers' pledgeable income relaxes their incentive compatibility constraint, suggesting that sellers' participation constraint is more restrictive when their moral hazard problem is not severe. I now proceed to derive the optimal contract that obtains for low, intermediate, and high levels of pledgeable income.

High pledgeable income. Consider first the case when sellers' pledgeable income is high such that at the optimal contract, sellers' incentive compatibility constraint (14) is slack and the optimal contract is determined by sellers' binding participation constraint (12) (Point A in Figure 1). This is the case if i) sellers' incentive constraint is downward-sloping, such that margin calls do not provide risk-prevention efforts, which occurs if $\mathcal{P} > (1-\pi)/q(\epsilon)$, and ii) the incentive constraint's intercept is above the one of the participation constraint, which occurs if $\mathcal{P} > \lambda/q(\epsilon)$. In summary, this optimal contract obtains if sellers' pledgeable income is high:

$$\mathcal{P} \geq \max\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\}. \tag{16}$$

The optimal payment y_g by non-defaulting sellers in aggregate state $(\underline{\theta}, g)$ is then determined by sellers' binding participation constraint (12) with no margin calls, that is with $\alpha = 0$, which yields $y_g^h = \pi\triangle\theta/p_{hg}$. It is straightforward to determine the remaining features of the optimal contract in this candidate equilibrium. Non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, b)$, y_b, is pinned down by equation (11): $y_b^h = \pi\triangle\theta/p_{hb}$. The total insurance payout Y to protection buyers in state $\underline{\theta}$ is determined by the CCPs' aggregate resource constraint (8): $Y^h = \pi\triangle\theta$. Lastly, the payout x sellers receive in the good aggregate state $\overline{\theta}$, which the CCP funds by collecting funds X from buyers, is determined by equation (10): $x^h = X^h = (1-\pi)\triangle\theta$. The insurance contract is actuarially fair since the expected transfer from protection sellers to protection buyers is zero: $(1-\pi)\,Y^h - \pi X^h = 0$

Notice that the optimal contract features $y_b^h > y_g^h$, that is, non-defaulting sellers' payment to the CCP is larger after a bad CCP-specific shock. Since the contract does not feature margin calls, sellers' limited liability constraint (7) requires that non-defaulting sellers' payoff from their risky asset $\tilde{R}$ is sufficient to honor the payment y_b^h to the CCP: $y_b^h \leq R$. Replacing y_b^h from above and rearranging shows that the contract satisfies sellers' limited liability constraint if $p_{hb}R > \pi\triangle\theta$. The limited liability condition effectively requires that pooling resources from all non-defaulting protection sellers in the worst possible aggregate state, $(\underline{\theta}, b)$, allows the CCP to fully insure protection sellers. This condition is stricter than assumption (A2), which guaranteed full insurance in the bilateral trading benchmark, since the CCP-specific shock allows reduces the CCP's available resources in the worst possible aggregate state.

Instead, if $\pi\triangle\theta > p_{hb}R$ holds, the limited liability constraint is binding and determines non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, b)$: $y_b^{LL} = R$. The remaining features of the contract are determined using $(\alpha = 0, y_b^{LL})$ as described above: equation (11) yields $y_g^{LL} = \frac{p_{hb}R}{p_{hg}} < y_b^{LL}$, equation (8) gives $Y^{LL} = p_{hb}R$, and lastly from equation (10) I obtain $x^{LL} = X^{LL} = \triangle\theta - p_{hb}R$. When sellers' limited liability constraint is binding the optimal contract is not actuarially fair since the expected transfer to a seller is positive:

$$\pi X^{LL} - (1-\pi)\,Y^{LL} = \pi\triangle\theta - p_{hb}R > 0.$$

Lastly, a key question of the analysis is whether central clearing can facilitate the provision of insurance with respect to the bilateral trading benchmark. In particular, does the optimal contract under central clearing deliver an insurance payout to protection buyers that is larger than in the bilateral benchmark, $\hat{Y}$? When protection sellers' limited liability constraint is not binding, that is if $\pi\triangle\theta \leq p_{hb}R$, it is trivial to show that the centrally cleared contract dominates:

$$Y^h = \pi \triangle\theta > \frac{\pi \triangle\theta}{p_h R} = \hat{Y}, \tag{17}$$

since assumption (A1) guarantees $p_h R > 1$. Instead, if the limited liability is binding the centrally cleared contract dominates the bilateral benchmark ($Y^{LL} \geq \hat{Y}$) only if $p_{hb} R \geq \pi\triangle\theta/(p_h R)$. Proposition 2 summarizes the optimal centrally cleared contract when protection sellers' pledgeable income is high.

Proposition 2 *(CCP Contract & High Pledgeable Income): In the optimal CCP contract when protection sellers' pledgeable income is high, $\mathcal{P} > \max\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\}$, sellers' participation constraint (12) binds and the limited liability constraint (7) is slack if $p_{hb}R > \pi\triangle\theta$. If $p_{hb}R \leq \pi\triangle\theta$ the limited liability constraint binds and central clearing improves upon the bilateral benchmark only if $p_{hb}R \geq \hat{Y}$. The insurance payout to protection buyers is*

$$Y^{h*} = \begin{cases} \pi\triangle\theta & \text{if } p_{hb}R > \pi\triangle\theta \\ p_{hb}R & \text{if } p_{hb}R \in \left[\frac{\pi\triangle\theta}{p_h R}, \pi\triangle\theta\right] \\ \hat{Y} & \text{if } p_{hb}R < \frac{\pi\triangle\theta}{p_h R} \end{cases}.$$

Figure 2 characterizes the insurance payout Y in the optimal centrally cleared contract described in Proposition 2. In particular, it shows how the size of protection buyers' risk exposure $\triangle\theta$ and the fraction of successful protection sellers after a bad CCP-specific shock p_{hb} affect the insurance payout. A ceteris paribus increase in p_{hb} leads to an increase in the insurance payout Y^{h*}. In particular, when few protection sellers' succeed after a bad CCP-specific shock, that is when p_{hb} is low, the CCPs' available resources collected from protection sellers are scarce. This limits the CCPs' ability to mutualize idiosyncratic counterparty risk and may even eliminate any gains from central clearing over the fully collateralized, bilateral trading benchmark. Similarly, for a given share of non-defaulting sellers' after a CCP-specific shock p_{hb}, a ceteris paribus increase in the size of buyers' risk-exposure $\triangle\theta$ implies that the CCP needs to collect more resources to fully insure buyers by setting $Y = Y^h$. As buyers' risk exposure increases, the CCP becomes unable to provide full insurance due to sellers' limited liability constraint. For relatively large risk exposures, the insurance implied by sellers' binding limited liability constraint does not provide any benefit over the fully collateralized bilateral trading benchmark.

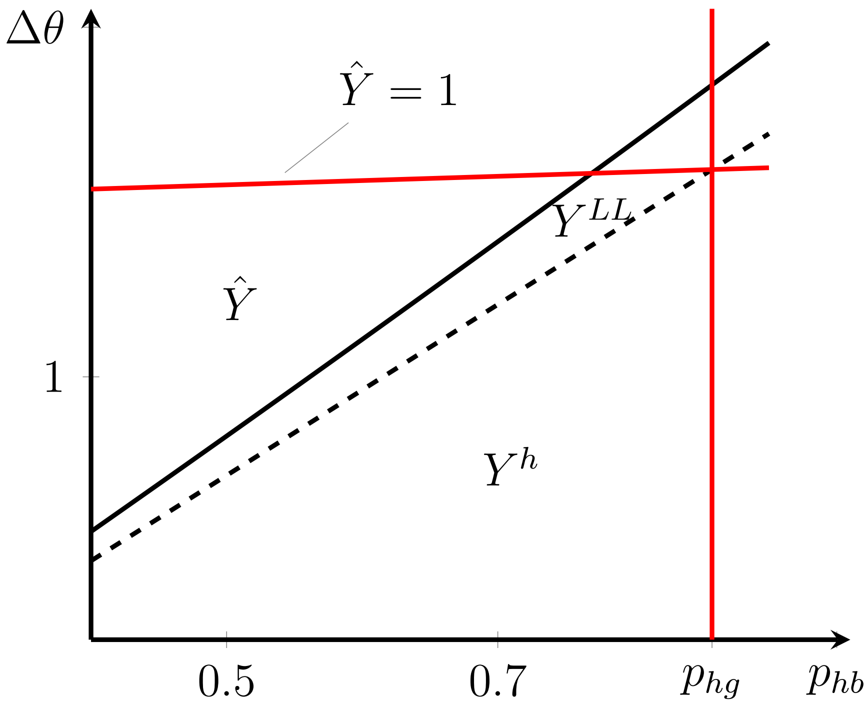

Figure 2: Insurance Payout Characterization under High Pledgeable Income.

This Figure characterizes the CCP's insurance payout Y^{h*} to protection buyers in aggregate state $\underline{\theta}$ when sellers' pledgeable income is high as a function of the fraction of successful sellers after a bad CCP-specific shock p_{hb} and buyers' aggregate risk exposure $\triangle\theta$. Red lines correspond to the parametric assumptions (A2), that is $\triangle\theta \leq p_h R/\pi$, and $p_{hb} < p_{hg}$.

Intermediate pledgeable income. Consider next the case in which at the optimal contract, protection sellers' participation constraint is slack and their incentive compatibility constraint is binding (Point B in Figure 1). This case obtains if sellers' incentive constraint is downward-sloping, that is, if $\mathcal{P} \geq (1-\pi)/q(\epsilon)$, and if the participation constraint's intercept $(y_g = \lambda/(1-\pi))$ is above the one of the incentive constraint $(y_g = q(\epsilon)\mathcal{P}/(1-\pi))$, which occurs if $\mathcal{P} \leq \lambda/q(\epsilon)$. In summary, this optimal contract obtains if sellers' pledgeable income is intermediate:

$$\mathcal{P} \in \left[\frac{1-\pi}{q\left(\epsilon\right)}, \frac{\lambda}{q\left(\epsilon\right)}\right]. \tag{18}$$

A necessary condition for this optimal contract to arise is $\lambda \geq 1-\pi$, which after replacing λ from equation (13) reduces to $\triangle\theta \geq p_{hg}/\pi$.[3] The optimal payment y_g by non-defaulting sellers in aggregate state $(\underline{\theta}, g)$ is then determined by sellers' binding incentive constraint (14) with no margin calls ($\alpha = 0$), which yields $y_g^m = q(\epsilon)\mathcal{P}/\left(1-\pi\right)$. The remaining features of the optimal contract can be recovered as in the case with high pledgeable income: Equation (8) determines the aggregate insurance payout to protection buyers in the bad aggregate state $\underline{\theta}$, $Y^m = p_{hg}y_g^m$, while equation (10) pins down the payout to protection buyers in the good aggregate state $\overline{\theta}$: $x = X = \triangle\theta - Y^m$. Non-defaulting sellers' payment to the CCP in state $(\underline{\theta}, s = b)$ is determined by equation (11): $y_b^m = p_{hg}y_g^m/p_{hb}$. The severity of sellers' moral hazard problem requires the optimal effort-inducing contract to offer them rents in order to elicit prudent risk management. Consequently, the contract is not actuarially fair as the net transfer to protection sellers

$$\pi X^m - \left(1-\pi\right)Y^m = \pi\triangle\theta - \frac{p_{hg}q(\epsilon)\mathcal{P}}{1-\pi} > 0$$

is positive since $\mathcal{P} < \lambda/q(\epsilon)$. The premium that protection sellers earn for providing insurance is increasing in both, the size of buyers' risk exposure $\triangle\theta$ and sellers' moral hazard problem (measured via a decline in pledgeable income $\mathcal{P}$).

As in the case of high pledgeable income, the contract features $y_b^m > y_g^m$ such that

[3]Instead, if $\triangle\theta \geq p_{hg}/\pi$ we have $\lambda < 1-\pi$. If sellers' pledgeable income satisfies $\mathcal{P} \in \left[\frac{\lambda}{q(\epsilon)}, \frac{1-\pi}{q(\epsilon)}\right]$, their incentive constraint is upward-sloping and has an intercept that is above the intercept of the participation constraint. Although margins help to provide risk-prevention efforts, the incentive constraint is strictly above the participation constraint and thus loose. The optimal contract is then determined by sellers' binding participation constraint and features no margin calls. Thus, this reduces to the setting of the contract with high pledgeable income and therefore features (y_g^h, α^h) along with the remaining features of the contract.

sellers' limited liability constraint (7) reduces to $y_b^m \leq R$, which can be written as

$$p_{hb}R \geq \frac{p_{hg}q(\epsilon)\mathcal{P}}{1-\pi}. \tag{19}$$

Notice that the right-hand side of this inequality is exactly Y^m, that is the CCP's insurance payout to protection buyers in state $\underline{\theta}$. If this condition is violated, the non-defaulting sellers' payment in aggregate state $(\underline{\theta}, b)$ is instead determined by their binding limited liability constraint (7): $y_b^{LL} = R$. The remaining features of the contract then coincide with the analysis of the binding limited liability constraint when sellers' pledgeable income is high. In particular, the aggregate insurance payout to protection buyers in state $\underline{\theta}$ is $Y^{LL} = p_{hb}R$, which dominates the bilateral benchmark only if $\pi\triangle\theta/(p_hR) \leq p_{hb}R$

Proposition 3 *(CCP Contract & Intermediate Pledgeable Income): In the optimal CCP contract when protection sellers' income is intermediate,* $\mathcal{P} \in \left[\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right]$*, sellers' incentive constraint* (14) *binds and the limited liability constraint is slack if condition* (19) *holds. Otherwise sellers' limited liability constraint* (7) *binds and central clearing improves upon the bilateral benchmark only if* $p_{hb}R \geq \hat{Y}$*. The insurance payout to protection buyers is*

$$Y^{m*} = \begin{cases} \frac{p_{hg}q(\epsilon)\mathcal{P}}{(1-\pi)} & \text{if } p_{hb}R > \frac{p_{hg}q(\epsilon)\mathcal{P}}{1-\pi} \\ p_{hb}R & \text{if } p_{hb}R \in \left[\frac{\pi\triangle\theta}{p_hR}, \frac{p_{hg}q(\epsilon)\mathcal{P}}{1-\pi}\right] \\ \frac{\pi\triangle\theta}{p_hR} & \text{if } p_{hb}R < \frac{\pi\triangle\theta}{p_hR} \end{cases}.$$

Figure 3 illustrates the characterization of the optimal centrally cleared contract described in Proposition 3 when sellers' pledgeable income is intermediate. It conveys much of the same intuition as Figure 2, namely that the benefits of central clearing are limited when buyers' risk exposure ($\triangle\theta$) is large, and when the share of non-defaulting protection sellers' after a bad CCP-specific shock (p_{hb}) is low. Additionally, however, Figure 3 shows that the parameter space in which the optimal contract with intermediate pledgeable income obtains and dominates the bilateral trading benchmark is quite restricted. This is because the contract obtains only if buyers' risk exposure is sufficiently large, that is if

$\triangle\theta \geq p_{hg}/\pi$.

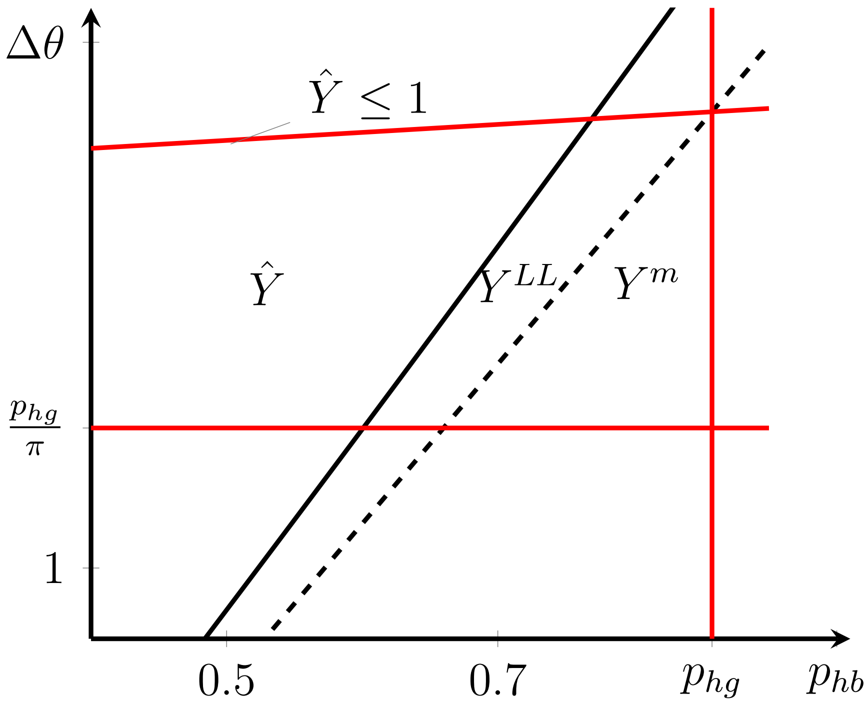

Figure 3: Central clearing when pledgeable income is intermediate.

This Figure characterizes the CCP's insurance payout Y^{m*} to protection buyers in aggregate state $\underline{\theta}$ when sellers' pledgeable income is intermediate as a function of the fraction of successful sellers after a bad CCP-specific shock p_{hb} and buyers' aggregate risk exposure $\triangle\theta$. Red lines correspond to the parametric assumptions (A2), that is $\triangle\theta \leq p_h R/\pi$, $p_{hb} < p_{hg}$ and $\lambda \geq 1 - \pi$, that is $\triangle\theta \geq p_{hg}/\pi$.

Low pledgeable income. Lastly, consider the case in which the optimal contract is determined by the intersection of protection sellers' participation and -incentive constraints (Point C in Figure 1). This case obtains if sellers' incentive constraint (14) is upward-sloping, that is, if $\mathcal{P} < (1-\pi)/q(\epsilon)$, and if the participation constraint's intercept is above the one of the incentive constraint, which occurs if $\mathcal{P} \leq \lambda/q(\epsilon)$. Hence this optimal contract obtains if and only if pledgeable income is low:

$$\mathcal{P} \leq \min\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\}.$$

When pledgeable income is low, margin calls help to provide risk-management efforts to protection sellers and thereby facilitate risk-sharing. The optimal margin call is determined by the intersection between sellers' participation and incentive-constraints, equations (12) and (14) respectievly, which yields:

$$\alpha^{l*} = \frac{p_{hg} \left(\lambda - q \left(\epsilon \right) \mathcal{P} \right)}{\left(\left(1 - \pi \right) p_h R - p_{hg} q \left(\epsilon \right) \mathcal{P} \right.} \tag{20}$$

Replacing the optimal margin call α^{l*} in protection sellers' participation constraint (12) yields non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, g)$: $y_g^l = \frac{\pi \triangle \theta}{p_{hg}} - \left[\frac{p_h R}{p_{hg}} - 1 \right] \alpha^{l*}$. Determining the remaining features of the optimal contract follows the same steps as previously and yields non-defaulting sellers' payment to the CCP in aggregate state $(\underline{\theta}, b)$

$$y_b^{l*} = \frac{\pi \triangle \theta}{p_{hb}} - \left[\frac{p_h R}{p_{hb}} - 1 \right] \alpha^{l*}, \tag{21}$$

the insurance payout to protection buyers

$$Y^{l*} = \pi \triangle \theta - \left(p_h R - 1 \right) \alpha^{l*}, \tag{22}$$

and finally the payments in the good aggregate state $\overline{\theta}$, $x^{l*} = X^{l*} = \left(1 - \pi \right) \triangle \theta + \left(p_h R - 1 \right) \alpha^{l*}$. Notice that equation (22) shows that the insurance payout to protection buyers Y^{l*} is decreasing in the size of margin calls α^{l*}. While margin calls are necessary to induce risk-prevention efforts by protection sellers, their usage is costly and thus limits the extent of risk-sharing that can be achieved.

Next, I verify if the optimal contract satisfies the feasibility constraint on margins (2). The margin call does not exceed sellers' unit endowment if $\alpha^{l*} \leq 1$. Replacing α^{l*} from equation (20) and rearranging yields $\triangle \theta \pi \leq p_h R$, which is satisfied by assumption (A2). Non-negativity of the margin call requires $\alpha^{l*} \geq 0$. The numerator of the margin call in equation (20) is positive since the initial assumption of low pledgeable income implies $\mathcal{P} \leq \lambda / q \left(\epsilon \right)$. Similarly, the denominator in equation (20) is positive since pledgeable income is low ($\mathcal{P} \leq (1 - \pi) / q \left(\epsilon \right)$) and assumption (A1) implies $p_h R > p_{hg}$, which combined imply

$$\mathcal{P} \leq \frac{p_h R}{p_{hg}} \frac{(1-\pi)}{q(\epsilon)}.$$

Finally, the contract must satisfy protection sellers' limited liability constraint (7), which requires that non-defaulting sellers' payoff from their risky asset and margin accounts is sufficient to honor the payment y_b^l in equation (21):

$$\frac{\pi \triangle \theta}{p_{hb}} - \left[\frac{p_h R}{p_{hb}} - 1\right] \alpha^{l*} \leq \left(1 - \alpha^{l*}\right) R + \alpha^{l*}.$$

Replacing a^{l*} from equation (20) and re-arranging shows that the limited liability condition under low pledgeable income reduces to $\pi \triangle \theta \leq p_h R$, which is satisfied by assumption (A2). Summarizing this analysis, assumptions (A1) and (A2) guarantee that the optimal contract under low pledgeable income satisfies the feasibility constraint on margin calls and protection sellers' limited liability constraint.

Having derived the optimal centrally cleared contract when sellers' pledgeable income is low, it remains to show whether the contract improves upon the bilateral benchmark, that is whether the insurance payout to protection buyers satisfies $Y^{l*} \geq \hat{Y}$. Replacing Y^{l*} from equation (22) and $\hat{Y}$ from Proposition 1 shows that the centrally cleared contract dominates if $\pi \triangle \theta - (p_h R - 1) \alpha^{l*} \geq \pi \triangle / (p_h R)$, which after re-arranging yields

$$(p_h R - 1) \left[\frac{\pi \triangle \theta}{p_h R} - \alpha^{l*}\right] \geq 0.$$

Recall that assumption (A1) guarantees $p_h R > 1$, so that the above condition is satisfied if the term in square brackets is positive. The first term in square brackets turns out to be the margin call in the bilateral benchmark $\hat{\alpha}$ described in Proposition 1. The condition thus states that the centrally cleared contract dominates the bilateral benchmark if it relies less on costly margin calls, that is if $\alpha^{l*} < \hat{\alpha}$. Replacing the optimal margin call under central clearing α^{l*} from equation (20) and re-arranging shows that this condition reduces to $p_h R \geq \pi \triangle \theta$, which is satisfied by assumption (**??**). Thus, the centrally cleared contract under low pledgeable income dominates the bilateral benchmark, as it resorts less to costly margin calls due to the benefits of diversifying idiosyncratic counterparty risk.

Proposition 4 summarizes the preceding discussion of the optimal CCP contract when protection sellers' pledgeable income is low.

Proposition 4 *(CCP Contract & Low Pledgeable Income): In the optimal CCP contract when protection sellers' income is low,* $\mathcal{P} \leq \min\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\}$*, margin calls are necessary to provide sellers with risk-prevention incentives. The optimal margin call* α^{l*} *is determined by equation (20) and supports the insurance payout to protection buyers* $Y^{l*} = \pi\triangle\theta - (p_h R - 1)\,\alpha^{l*}$*. Sellers' limited liability constraint (7) is slack and the centrally cleared arrangement dominates the bilateral benchmark.*

3.1 CCP design & optimal clearing arrangements

A crucial feature of the optimal centrally cleared contract, regardless of protection sellers' pledgeable income, is that non-defaulting sellers' transfer to the CCP in aggregate state $\underline{\theta}$ is larger after a bad CCP-specific shock than after a favorable CCP-shock, that is the centrally cleared contract features $y_b^* > y_g^*$. Insuring protection buyers against the CCP-specific shock requires successful sellers to absorb some of the default losses induced by the shock. Effectively, successful sellers pay larger transfers after a bad CCP-shock to make up for the fact that there are fewer sellers making these payments (p_{hb} as opposed to p_{hg} of all sellers).

This feature of the model provides a rationale for the current design of CCPs' "loss allocation rules" (so-called default waterfalls). In practice, clearing members have to contribute not only margin deposits to the CCP but are also required to invest into the so-called "default fund". In case the CCP faces a counterparty default in excess of those covered by the defaulter's resources (margins and default fund) it will draw upon the default fund contributions of *non-defaulting* members. These practices are in line with the predictions of the model: as defaults increase, the CCP's only way to mutualize these losses is by requesting more resources from non-defaulting clearing members.

However, protection sellers' limited liability constraint (7) restricts the extent to which the CCP can mutualize losses from clearing member defaults. As the insurance payouts in the contract with high and intermediate pledgeable income, described in Propositions 2 and 3 respectively, showed is that the CCP's insurance provision is not restricted by sellers' limited liability constraint if the share of non-defaulting clearing members after an adverse CCP shock (p_{hb}) is sufficiently high. If too many sellers default after an adverse CCP shock (p_{hb} is low), completely mutualizing these losses would require very large payments from non-defaulting sellers, which violates their limited liability constraint.

Figure 4 illustrates how changes in protection sellers' pledgeable income $\mathcal{P}$, and the likelihood of the adverse CCP-specific shock ϵ, affect which type of optimal contract obtains in equilibrium. As sellers' moral hazard problem becomes more severe, and pledgeable income consequently declines, inducing effort by protection sellers requires the CCP to pay them increasing rents at the expense of decreases in the insurance payout to protection buyers. Moreover, the figure illustrates that a ceteris paribus increase in the likelihood of an adverse CCP-specific shock shifts the equilibrium towards the optimal contract with intermediate- and low pledgeable income. Formally, this is because an increase in ϵ decreases $q(\epsilon)$, which shifts protection sellers' incentive constraint (14) downward and increases its slope coefficient. Intuitively, an increase in ϵ reduces the success probability p_h of protection sellers' risky asset $\tilde{R}_j$, thereby reducing the opportunity cost of margin calls $(p_h R - 1)$. As the efficiency cost of margin calls decrease, they are more likely to arise in equilibrium.

The preceding discussion and the optimal insurance payouts described in Propositions 1-4 highlight that the size of protection buyers' risk exposure $\triangle\theta$ and the likelihood of an adverse CCP-specific shock ϵ are crucial determinants of the insurance payout to protection buyers in the bilateral trading benchmark and under central clearing. Figure 5 depicts how these two parameters determine the optimal clearing arrangement.

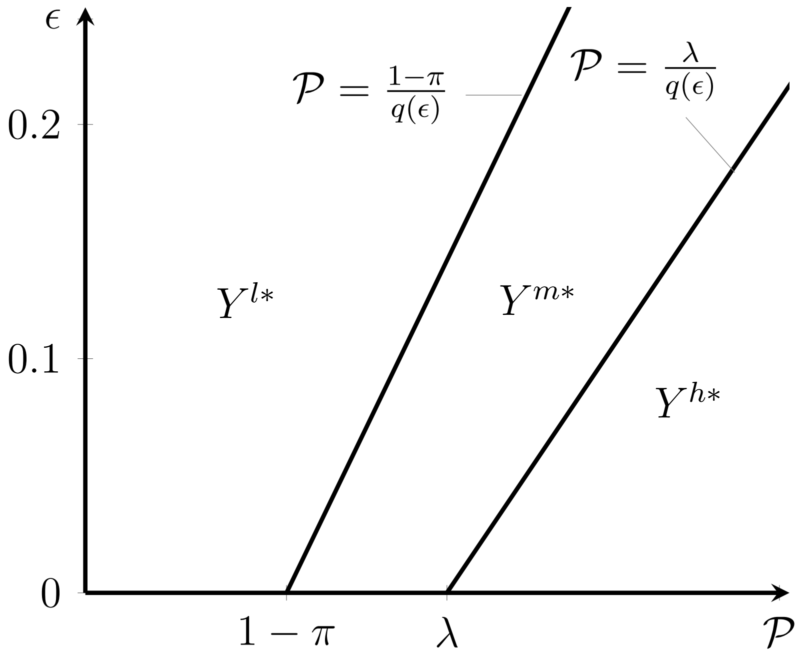

Figure 4: Characterization of the optimal insurance payout Y

This figure illustrates how protection sellers' pledgeable income $\mathcal{P}$ and the likelihood of an adverse CCP-specific shock ϵ determine the optimal insurance payout Y to protection buyers in aggregate state $\underline{\theta}$. Y^{h*}, Y^{m*}, and Y^{l*} correspond respectively to the insurance payout when sellers' pledgeable income is high, intermediate, and low.

The figure illustrates that a ceteris paribus increase in buyers' risk exposure moves the economy closer to an equilibrium in which the optimal bilateral contract dominates centrally cleared arrangements. Due to the correlation of protection sellers' outcomes at a given CCP, the CCP has to insure buyers not only against their own risk $\tilde{\theta}$, but also against the CCP-specific shock s. It does so by mutualizing losses across protection sellers ($y_b > y_g$). However, as buyers' risk exposure grows the required degree of loss mutualization becomes too large and violates non-defaulting sellers' limited liability constraint. This can lead to situations in which the degree of insurance provided by the CCP is so restricted that it ceases to improve upon the bilateral trading benchmark. Similarly, as the likelihood of an adverse CCP-specific shock increases, the fully collateralized bilateral contract is more likely to be optimal in equilibrium. This is due to the decrease in the

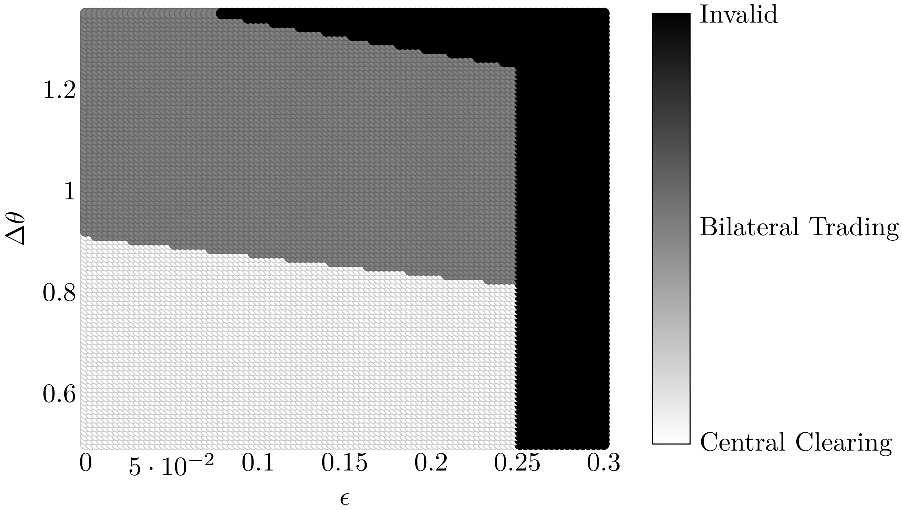

Figure 5: Optimal clearing arrangement

This figure depicts how the size of protection buyers' risk exposure $\triangle\theta$ and the likelihood an adverse CCP-specific shock affect the optimal clearing arrangement. Parameter configurations for which central clearing is optimal are shaded in light grey, while those in which bilateral trading is optimal are shaded in dark grey. Parameter configurations which violate assumptions (A1) or (A2) are shaded black.

opportunity cost of margin calls, which reduce the efficiency loss entailed in the fully collateralized bilateral contract.

These results have important policy implications. Particularly, central clearing arrangements should be favored in products that are less risky and when the degree of correlation in the outcomes of the institutions trading them is low. This suggests that the degree of correlation across CCPs' clearing members' portfolios should be a key factor to analyze for regulators, as the extent of margin calls in CCP contracts is intimately tied to it.

4 Incentive problems at the CCP level

In this section I analyze how the introduction of moral hazard problems at the CCP level affect the provision of insurance and risk sharing. I show that, when diligent and costly monitoring efforts by the CCP are needed to induce effort by protection sellers, CCPs themselves must be incentivized to contribute to the reduction of counterparty risk. While a combination of clearing fees and CCP capital can provide these incentives, moral hazard problems at the CCP level limit the scope for risk sharing and lower the provision of insurance to protection buyers, as they ultimately bear the cost of providing monitoring incentives to the CCP.

The incentive problem at the CCP is set up similarly to the problem of the monitoring intermediary in Holmström and Tirole (1997). The CCP can monitor protection sellers at a non-verifiable, pecuniary cost $\phi > 0$, which reduces protection sellers' private benefits from $\tilde{B} > B$ to B and increases their pledgeable income $\mathcal{P}$. The increase in pledgeable income relaxes sellers' incentive compatibility constraint (14) and thus allows for potentially higher insurance payments Y.[4]

In line with the previous analysis, I focus on optimal contracts that induce monitoring efforts by the CCP and risk-prevention efforts by sellers. Importantly, the CCP's monitoring choice could be observable *ex-post* through the mass of defaulting protection sellers: conditional on the CCP-specific shock s, a monitoring CCP will not see more than $1 - p_{hs}$ of all protection sellers fail, whereas absent monitoring this fraction is $1 - p_{ls}$. In other words, conditional on s, there is a clear link between the (observable) overall performance of protection sellers and the CCP's effort choice. Therefore, if the CCP-specific shock s was observable, a shirking CCP could be punished ex-post whenever it does not monitor. To make the problem of providing monitoring incentives to the CCP non-trivial I consider the case in which s is unobservable and

$$p_{hb} = p_{lg}. \tag{A4}$$

Under this assumption it is impossible to distinguish whether the CCP monitored diligently and a bad shock arrived ($s = b$) or if the CCP shirked and was lucky to receive a good shock ($s = g$). Hence, ex-post there are only three distinguishable states of CCP

[4] This holds unless pledgeable income is already large so that the incentive constraint is slack for $\tilde{B}$ and the binding participation constraint determines the insurance payment.

performance (measured through the mass of non-defaulting protection sellers): either p_{hg}, $p_{hb} = p_{lg}$ or p_{lb} of all sellers succeed. The optimal contract will be designed such that a large share (p_{hg}) of succeeding protection sellers only occurs when the CCP monitored sellers and ensured prudent risk-management. Conversely, when only p_{lb} of all protection sellers succeed the CCP must have slacked.

Providing the CCP with monitoring incentives requires the optimal contract to specify a compensation scheme that is contingent on the observable state of the economy, and therefore the performance of clearing members. For this purpose I extend the original contracts by the CCP with buyers and sellers in two ways: first, I assume that the CCP is required to use some of its own capital K to guarantee the payment of Y to protection buyers when the fraction of defaulting sellers is larger than $1 - p_{hg}$. Hence the CCP keeps its capital K in the bad aggregate state $\underline{\theta}$ only when it induces effort by protection sellers and is not hit by a bad CCP-specific shock ($s = b$). The capital is raised at date 0 from the CCP's owners whose opportunity cost of capital $R_k > p_h R$ (from here on I refer to R_k as the CCP's funding cost). The possibility of losing its capital provides the CCP with incentives to adopt a risk management framework that minimizes the potential losses it faces (see Carter and Garner (2015)). Due to the CCP's moral hazard problem, providing monitoring incentives to the CCP requires its owners' to have "skin in the game".

Second, as a means to compensate the CCP for the cost of raising (and possibly losing) costly capital, I introduce a clearing fee F that protection buyers pay to the CCP. While the clearing fee could also be fully or partially paid by protection sellers, assuming that protection buyers are solely responsible for paying the fee has the convenient property that sellers' participation and incentive-constraints derived in the previous section remain unchanged. This assumption thus greatly simplifies the analysis.

Depending on the performance of its clearing members, the CCP passes the funds collected via the clearing fee on to either the CCP owners or protection buyers. I assume that the CCP compensates its owners for putting their capital at risk in all but the worst possible aggregate state $(\underline{\theta}, b)$. As the following analysis shows, both the clearing fee and

CCP owners' capital at risk provide monitoring incentives to the CCP. Consequently, assuming that CCP owners receive the clearing fee in most aggregate states is made so as to not overemphasize the incentive role of CCP capital.

In summary, the (residual) transfer to the CCP's owners when exerting monitoring efforts is

$$\tau\left(\tilde{\theta}, s\right) = \begin{cases} 0 & \text{if } \tilde{\theta} = \underline{\theta}, s = b \\ F + K & \text{otherwise} \end{cases}.$$

Since the clearing fee and the CCP's capital may be used to honor payments to protection buyers in the worst possible aggregate state $(\underline{\theta}, b)$, the the insurance payout by the monitoring CCP is subject to a modified aggregate resource constraint:

$$Y = p_{hg} y_g + (1 - p_{hg})\, \alpha = p_{hb} y_b + (1 - p_{hb})\, \alpha + K + F. \tag{23}$$

The aggregate resource constraint (23) highlights that the CCP's capital K and the collected clearing fee F are used to finance the insurance payout Y in the worst aggregate state $(\underline{\theta}, b)$. Hence, from the perspective of protection buyers, CCP's capital is a substitute for resources provided via payments from non-defaulting sellers (y_b) to the CCP and defaulting sellers' margin accounts (α). It is important, however, to keep in mind that margin accounts provide resources at the cost of the risk-free rate whereas CCP capital comes at the cost of the CCPs' funding cost R_k.

In summary, the addition of the CCP's moral hazard problem leads to two changes in the optimal contracting problem: the design of the optimal compensation scheme $\{F, K\}$ for the CCP and the replacement of the aggregate resource constraint (8) by the modified resource constraint (23).

4.1 The optimal CCP compensation scheme

I now derive the optimal compensation scheme for the CCP to guarantee effort-provision by protection sellers as well as the CCP. Thus, in the following I derive the transfers

$\{F, K\}$ that induce monitoring by the CCP. I assume that competitive pressures in the CCP industry lead the CCP to break even when inducing effort by protections sellers. Since the CCP's capital is at risk of being used for insurance payments to protection buyers in state $(\underline{\theta}, b)$, CCP owners have to be appropriately compensated for putting their capital at risk. The resulting participation constraint of CCP owners,

$$\pi (F + K) + (1 - \pi) (1 - \epsilon) (F + K) \geq R_k K + \phi,$$

highlights that the expected transfer to the CCP owners on the left-hand side must be sufficiently high to compensate for the opportunity cost of investing their funds into alternative investment projects and the cost of monitoring on the right-hand side. The participation constraint highlights that the CCP's owners earn an agency rent, that is a compensation that is strictly above the cost of exerting effort, due to the unobservability of their monitoring efforts. Solving the CCP's participation constraint for the clearing fee F yields

$$F \geq \frac{\phi}{[\pi + (1 - \pi) (1 - \epsilon)]} + \left[\frac{R_k}{[\pi + (1 - \pi) (1 - \epsilon)]} - 1 \right] K \tag{24}$$

The above inequality highlights that an increase in the CCP's cost of monitoring ϕ as well as an increase in CCP capital K tighten the CCP's participation constraint. Intuitively, incurring higher monitoring cost and/or putting higher levels of capital at risk make running the CCP (and monitoring its clearing members) more costly for its owners, and consequently render their outside option more appealing. Therefore, the optimal clearing fee F will have to increase with both variables.

Next, the optimal contract must ensure that it is in the interest of the CCP to monitor its clearing members. This leads to the CCP's incentive compatibility constraint

$$\pi (F + K) + (1 - \pi) (1 - \epsilon) (F + K) - \phi \geq \pi (F + K) .$$

The CCPs' incentive constraint highlights that the expected transfer to the CCPs' owners under monitoring, net of the monitoring cost, (left-hand side) is higher than the expected transfer without monitoring efforts (right-hand side). Absent monitoring efforts, protection sellers' private benefits are high and they consequently do not exert risk-prevention efforts. Therefore, the share of non-defaulting protection sellers will at most be $p_{hb} < p_{hg}$, so that CCP owners will lose their capital if buyers' risky asset pays only $\underline{\theta}$, regardless of the realization of the CCP-specific shock s. Going forward, it will provide convenient to re-arrange the CCP's incentive constraint:

$$F + K \geq \frac{\phi}{(1-\pi)(1-\epsilon)}. \tag{25}$$

The CCP's incentive constraint shows that both CCP capital and the clearing fee are tools to provide monitoring incentives. This highlights the role of CCP capital in not only guaranteeing contractually promised payments to clearing members but also to efficiently re-allocate counterparty risk.[5] Comparing the CCPs' participation constraint (24) with the incentive constraint (25) highlights the effects of CCP owners' "skin in the game": while increasing the amount of CCP capital K provides monitoring incentives to the CCP, thereby relaxing its' incentive constraint above, it also makes it more costly to guarantee CCP owner's participation in the clearing arrangement.

The objective of the optimal contract is to maximize protection buyers utility while guaranteeing risk-prevention efforts by protection sellers and monitoring by the CCP. Since protection buyers bear the cost of the clearing fee F, it is optimal to set the lowest clearing fee compatible with the CCPs' participation and incentive constraints. Similarly, since it is costly to raise CCP capital, the optimal contract will not require more capital than strictly needed to satisfy the CCPs' constraints (24) and (25). Intuitively, maximizing protection buyers' utility requires to minimize the CCPs' compensation while guaranteeing its participation and monitoring incentives. Therefore, I treat both the CCP's participation constraint (24) and incentive constraint (25) as binding. This leads to a simple system of two linear equations in the two unknowns K and F. The following

[5] See Pirrong (2011) for further discussions.

proposition highlights the optimal CCP compensation scheme obtained by solving this system of equations, and some of the solution's key properties.

Proposition 5 *(CCP Compensation Scheme): In the optimal contract with risk-prevention efforts by protection sellers and monitoring by the CCP, the level of CCP capital is*

$$K^* = \frac{\pi}{R_k} \frac{\phi}{(1-\pi)(1-\epsilon)}. \tag{26}$$

and the clearing fee paid by protection buyers is

$$F^* = \left(\frac{R_k - \pi}{R_k}\right) \frac{\phi}{(1-\pi)(1-\epsilon)}. \tag{27}$$

Both K^* and F^* are increasing in the CCP's cost of monitoring ϕ and the likelihood of a bad CCP-specific shock ϵ. An increase in the CCP's funding cost R_k leads to a decrease in CCP capital K^* and an increasing in the clearing fee F^*.

They properties of the optimal CCP compensation scheme described in Proposition 5 are intuitive and highlight the dual role of CCP capital as a tool to provide monitoring incentives and prefunded resources to absorb clearing member defaults: As the CCP's moral hazard problem becomes more severe (measured by an increase in monitoring cost ϕ), providing monitoring incentives to the CCP becomes more costly, which warrants an increase in the expected transfer to the CCP via clearing fees. Similarly, CCP owners' need to have more "skin in the game", that is, the amount of their capital at risk must increase when the CCP's incentive problem is severe.

An increase in the probability of the bad CCP-specific shock ϵ increases the likelihood of the CCP facing a high share $(1 - p_{hb})$ of defaulting protection sellers. Sustaining insurance payments to protection buyers then requires the CCP to have an increasing amount of pre-funded resources in the form of clearing fees and CCP capital. Lastly, as the CCP's funding cost increases, a given amount of CCP capital becomes more expensive. Consequently, less CCP capital is required to provide monitoring incentives to the CCP when its funding cost (R_k) is high. Meanwhile the clearing fee has to increase in order to keep the expected transfer to the CCP owners constant when its funding cost increase.

4.2 Optimal contracts when CCPs face moral hazard

It remains to be shown how the CCP's moral hazard problem affects the optimal central clearing arrangement derived in section 3. Since protection buyers always pay the clearing fee F, the full insurance condition (1) of the contract remains unchanged

$$\bar{\theta} - X - F = \underline{\theta} + Y - F \quad \Leftrightarrow \quad \triangle\theta = Y + X.$$

Further, since neither CCP's capital nor the clearing fee directly interfere with the problem of protection sellers, their participation and incentive constraints, equations (12) and (14) respectively, are unaffected.

The key difference in the optimal contracting problem is the replacement of the aggregate resource constraint (8) by the modified resource constraint (23), which accounts for the CCP's payments to protection buyers via its capital contribution and fee repayment in the aggregate state $(\underline{\theta}, b)$. From the perspective of protection buyers, these payments are substitutes for resources provided by protection sellers. Failing to adjust the optimal contracts derived in section 3 would therefore fail to provide full insurance to protection buyers as their consumption levels would be higher in the adverse aggregate state $(\underline{\theta}, b)$. Consequently, the payments of non-defaulting protection sellers to the CCP in state $(\underline{\theta}, b)$, y_b, have to be reduced to equalize buyers' utility across all states of nature. I begin by solving the modified aggregate resource constraint (23) for y_b to find

$$\hat{y}_b^* = \frac{1}{p_{hb}} \left[Y^* - (1 - p_{hb})\, \alpha - (K^* + F^*) \right].$$

For $K^* + F^* = 0$ this reduces to equation (11), which determined the payment by non-defaulting sellers to the CCP in state $(\underline{\theta}, b)$ in the baseline model, y_b^*. Consequently, in the optimal contract when the CCP faces moral hazard, non-defaulting sellers' payment to the CCP is

$$\hat{y}_b^* = y_b^* - \frac{K^* + F^*}{p_{hb}} = y_b^* - \frac{1}{p_{hb}} \frac{\phi}{(1-\pi)(1-\epsilon)}.$$

The reduction in non-defaulting sellers' payment to the CCP after a bad CCP-specific shock facilitates the provision of insurance to protection buyers. Recall that in the optimal contract when sellers' pledgeable income is high (Proposition 2) the insurance payout implied by sellers' binding participation constraint may require a payment from non-defaulting sellers to the CCP after a bad CCP-shock y_b^h which violates their limited liability constraint (7). If this is the case, sellers' binding limited liability constraint restricts the insurance payout Y to protection buyers. Thus, the reduction in non-defaulting sellers' payment implied by the CCP compensation scheme described in Proposition 5 expands the parameter space over which sellers' limited liability constraint does not limit the provision of insurance to protection buyers Y. To make this point formally, note that sellers' limited liability constraint now reads $\hat{y}_b^* \leq R$, which after replacing $y_b^h = \pi \triangle \theta$ from the analysis in section 3 yields

$$p_{hb} R \geq \pi \triangle \theta - (K^* + F^*) .$$

Clearly an increase in CCP capital or the clearing fee relaxes protection sellers' limited liability constraint. The same reasoning applies to the optimal contract when sellers' pledgeable income is intermediate (Proposition 3), in which the insurance payout is determined by sellers' binding incentive constraint up until the point where the implied payment by non-defaulting sellers after a bad CCP shock y_b^m violates their limited liability constraint.

Interestingly, the CCPs' moral hazard problem does not affect the provision of insurance when protection sellers' pledgeable income is low. Recall that the corresponding optimal contract described in Proposition 4 highlights that sellers' moral hazard problem warrants the use of costly margin calls to provide sellers with risk-prevention incentives. The use of costly margins limits the insurance payout to protection buyers to such an extent that the implied payment by non-defaulting sellers' after a bad CCP shock y_b^l never violates sellers' limited liability constraint. Consequently, relaxing sellers' limited liability constraint with the optimal CCP compensation scheme has no impact on the insurance provision to protection buyers. Proposition 6 summarizes the preceding analysis and highlights how the optimal CCP compensation scheme affects the insurance provided by the central clearing arrangement.

Proposition 6 *(Insurance Provision & CCP Moral Hazard) In the optimal contract with risk-prevention efforts by protection sellers and monitoring by the CCP, the compensation scheme (F^*, K^*) described in Proposition 5 facilitates the provision of insurance unless protection sellers' pledgeable income is low, $\mathcal{P} < \min\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\}$, in which case the optimal contract in Propositon 4 remains unchanged.*
If sellers' pledgeable income is high, $\mathcal{P} > \max\left\{\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right\}$, the insurance payout is

$$\hat{Y}^{h*} = \begin{cases} \pi\triangle\theta & \text{if } p_{hb}R > \pi\triangle\theta - (K^* + F^*) \\ p_{hb}R & \text{if } p_{hb}R \in \left[\frac{\pi\triangle\theta}{p_h R}, \pi\triangle\theta - (K^* + F^*)\right] \\ \frac{\pi\triangle\theta}{p_h R} & \text{if } p_{hb}R < \frac{\pi\triangle\theta}{p_h R} \end{cases},$$

whereas if pledgeable income is intermediate, $\mathcal{P} \in \left[\frac{1-\pi}{q(\epsilon)}, \frac{\lambda}{q(\epsilon)}\right]$, the insurance payout is

$$\hat{Y}^{m*} = \begin{cases} \frac{p_{hg}q(\epsilon)\mathcal{P}}{(1-\pi)} & \text{if } p_{hb}R > \frac{p_{hg}q(\epsilon)\mathcal{P}}{1-\pi} - (K^* + F^*) \\ p_{hb}R & \text{if } p_{hb}R \in \left[\frac{\pi\triangle\theta}{p_h R}, \frac{p_{hg}q(\epsilon)\mathcal{P}}{1-\pi} - (K^* + F^*)\right] \\ \frac{\pi\triangle\theta}{p_h R} & \text{if } p_{hb}R < \frac{\pi\triangle\theta}{p_h R} \end{cases}.$$

5 Implications

The results in this paper provide a theoretical guideline for the design of CCP's loss allocation rules ("default waterfalls"). Section 3 showed that when protection sellers face moral hazard problems, margin calls serve not only as prefunded financial resources, but also as a disciplining device to provide sellers with prudent risk-management incentives. Similarly, section 4 highlighted that when CCPs face moral hazard problems themselves, using some of the CCPs' own capital to guarantee insurance payouts fulfils a similar dual role as a financial resource and a device to provide monitoring incentives to the CCP.

Importantly, incentivizing protection sellers and the CCP requires margin deposits and CCP capital to be at the first stage of the default waterfall. Resources from non-defaulting protection sellers (y_s) are only used as a residual contributions to make up for the difference between the insurance payout Y and the combination of margins and CCP capital. The results thus support the most common design of default waterfalls, in which CCPs draw upon non-defaulting members resources only if the combination of defaulting members' margins and CCP capital is insufficient to cover outstanding losses (Carter and Garner (2015) and Pirrong (2011)).

Interestingly, the model suggests that, depending on the severity of the incentive problem at the CCP, insuring protection buyers against the CCP-specific shock may not require mutualizing sellers' losses across states of nature. In other words, severe incentive problems at the CCP (high ϕ) may require the CCP to have substantial skin in the game (high K) such that non-defaulting protection sellers' payment to the CCP after an adverse CCP-shock is not larger than after a favorable CCP-shock, that is $y_g^* > y_b^*$. To see this formally, note that the moral hazard facing CCPs' aggregate resource constraint (23) implies

$$y_b^* = \frac{1}{p_{hb}} \left[p_{hg} y_g^* - (p_{hg} - p_{hb})\, \alpha^* - (K^* + F^*) \right]$$

The discussion of the optimal CCP compensation scheme (K^*, F^*) highlighted that an increase in the CCP' funding cost, for example because regulatory requirements make CCP capital more scarce, lead to a decrease in the optimal level of CCP capital and an increase in the optimal clearing fee. The increased operating cost of the CCP is ultimately passed on onto protection buyers as they have to pay a higher clearing fee. This result suggests that ill-designed capital regulation for CCPs may ultimately make protection buyers worse off. Figure 6 illustrates this potential adverse effect by comparing the net transfer $(Y^* - F^*)$ that buyers receive under various CCP arrangements with their outcome under the bilateral trading benchmark. Figure 6 highlights that the cost of providing monitoring incentives to the CCP is ultimately borne by protection buyers who pay the clearing fee F. The moral hazard problem at the CCP clearly diminishes the value of central clearing arrangements to protection buyers. Hence, the results indicate that the provision of insurance by the CCP is limited not only by protection sellers' moral hazard problem but also by the incentive problem of the CCP. Figure 6 shows that central clearing is beneficial for protection buyers only when both moral hazard problems are mild and the funding cost of CCP's is relatively low.

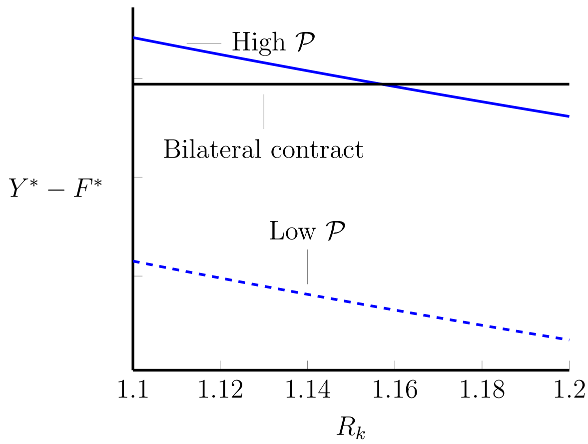

Figure 6: Net payments to protection buyers

This Figure illustrates the net payments $(Y^* - F^*)$ from the CCP to protection buyers in the bad aggregate state $\underline{\theta}$ for two different levels of pledgeable income $\mathcal{P}$: high (blue, solid line) and low (blue, dashed line). It also illustrates the payment to protection buyers under the bilateral arrangement $\hat{Y}$ (black, solid line).

6 Conclusion

I analyze optimal contracts in the context of central clearing arrangements in derivative markets. The results highlight that central clearing arrangements facilitate the provision of insurance by mutualizing idiosyncratic counterparty risk. However, moral hazard problems for protection sellers as well as CCPs limit risk-sharing. I show that margin requirements are necessary to reduce counterparty risk only when protection sellers face severe moral hazard. Similarly, the results suggest that CCPs must be required to absorb some of the losses they face with their own capital. When CCPs have "skin in the game", they have incentives to monitor their clearing members and induce prudent risk-management practices.

Importantly, I show how shocks that induce correlation in the asset returns of clearing members affect the degree of insurance provided by CCPs, and the design of the optimal clearing arrangement. In the presence of correlated clearing member defaults, CCPs mutualize losses from such defaults across clearing members by requesting additional funds from non-defaulting members. The results thus provide a theoretical basis for the design of CCP loss allocation rules requiring a default fund contribution by clearing members, which may be used to mutualize losses when default rates among clearing members are high. Moreover, I show that a moderate degree of correlation among clearing members' outcomes can severely limit the degree of risk-sharing that CCPs can achieve and may eliminate all benefits from central clearing.

The results do not indicate any adverse effect of CCPs' capital contributions on clearing members risk-prevention incentives. Carter and Garner (2015) argued that clearing members may be more inclined to gamble on their own assets when the CCP has sufficient skin in the game. The results presented here do not predict this effect since the CCP's capital is required to induce monitoring by the CCP, which in turn makes it *easier* to induce proper risk-management by clearing members.

When the CCP is required to put some of its own capital at risk it is necessary to compensate the providers of CCP capital. I highlight that a clearing fee can provide the necessary revenues. However, depending on the strength of the moral hazard problem that the CCP faces, the required clearing fees may be so high that insurance buyers do not benefit from the introduction of central clearing. However, there may be additional positive effects arising from clearing fees that are outside of this model. In particular, the clearing fee can be interpreted as an entry requirement to join central clearing arrangement, that mitigates adverse selection problems since only sufficiently capitalized and liquid insurance traders can join. As Pirrong (2011) and CPSS (2010) highlighted, such entry requirements reduce the heterogeneity of clearing members along with various problems arising due to heterogeneity. Finally, I show that the implied incentive-based capital requirements for CCPs decrease with the cost of CCP capital.

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