

Liquidity Regulation and Fire Sales in Mutual Funds

Dominic Cucic[†]

Danmarks Nationalbank

March 26, 2025

Abstract

We model interactions between investors, mutual funds, and liquidity providers to compare regulatory approaches for managing fund liquidity risk. Investor redemptions trigger asset sales by mutual funds, potentially causing fire sales when endogenous market liquidity becomes scarce. A pecuniary externality arises as liquidity providers fail to internalize price impacts, resulting in inefficiently low market liquidity. Liquidity requirements increase system-wide resilience despite partially crowding out private liquidity provision. In contrast, redemption restrictions undermine liquidity providers' incentives, potentially increasing fire-sale risk and reducing welfare. Effective regulation depends on its general equilibrium impact on liquidity providers' ex-ante incentives, not merely its direct effect on funds.

Keywords: Mutual funds, fire sales, financial regulation, market liquidity

JEL Classification: D62, G11, G23, G38

[†]Danmarks Nationalbank, Langelinie Allé 47, DK-2100 Copenhagen. Email: dc@nationalbanken.dk. I am grateful to my thesis advisors Rafael Repullo and Javier Suarez for their excellent guidance and continuous support. The paper has benefitted from helpful discussions by David Martinez-Miera, Tano Santos, Sergio Vicente, and Fabiana Gomez. I also wish to thank seminar participants at CEMFI, Copenhagen Business School, the 7th Emerging Scholars in Banking and Finance conference, the BCU-RIDGE Financial Stability Workshop, and Danmarks Nationalbank for valuable comments. Funding from the Santander Research Chair at CEMFI and Spain's Ministerio de Ciencia, Innovación y Universidades (PRE2018-084535) is gratefully acknowledged. Any remaining errors are my own. Views expressed herein are those of the author and do not necessarily reflect those of Danmarks Nationalbank.

1 Introduction

The exponential growth of the open-ended mutual fund sector, now managing over USD30 trillion globally, has created a significant structural vulnerability in financial markets. These funds offer daily liquidity to shareholders while increasingly holding illiquid assets, creating a liquidity mismatch that poses systemic risks. As noted by the Federal Reserve, "fixed-income mutual funds continue to be vulnerable to large, sudden redemptions, and sizable outflows can still lead to a deterioration in market liquidity of underlying assets" ([Federal Reserve Board, 2020](#)). The COVID-19 market turmoil of March 2020 provided a stark illustration of these concerns, as corporate bond funds experienced unprecedented outflows that exacerbated market illiquidity and necessitated extraordinary central bank interventions ([Haddad, Moreira and Muir, 2021](#); [Kargar et al., 2021](#); [Falato, Goldstein and Hortaçsu, 2021](#)).

In response to these vulnerabilities, policymakers have advanced competing regulatory approaches to enhance mutual fund resilience. The debate primarily centers on two contrasting interventions: (1) ex-ante liquidity requirements that mandate minimum holdings of cash and liquid securities, and (2) ex-post redemption restrictions such as gates or suspension mechanisms that limit investor withdrawals during stress periods. Thus far, the literature lacks a comprehensive theoretical framework to compare these approaches and evaluate their general equilibrium implications for market liquidity and financial stability.

This paper develops an equilibrium model to systematically compare regulatory approaches for managing liquidity risk in open-ended mutual funds. Our analysis reveals a critical insight often overlooked in current policy discussions: regulations targeting the same risk can have dramatically different welfare implications due to their distinct impacts on liquidity providers' incentives to supply liquidity to asset markets. We show that ex-ante resilience-building measures such as liquidity requirements generally outperform ex-post restrictions on investor redemptions because they preserve market functioning while addressing the underlying inefficiency.

Our model features three types of agents: investors, mutual funds, and liquidity providers. Investors place their capital in mutual funds, which invest in illiquid long-term assets. At an interim date, investors may redeem shares to pursue short-term investment opportunities, forcing mutual funds to sell assets in the secondary market. Liquidity providers, who are vital to market functioning, make ex-ante decisions about how much liquidity to hold in anticipation of potential fire sales. This structure allows us to analyze the general equilibrium effects of regulatory interventions on market liquidity, asset prices, and welfare.

We show that the competitive equilibrium exhibits a pecuniary externality arising from cash-in-the-market pricing ([Allen and Gale, 1994, 2005](#)): liquidity providers fail to internalize how their ex-ante liquidity decisions affect asset prices and, consequently, investors' ability to fund productive investments. This externality results in inefficiently low market liquidity and excessive fire sales, providing a clear rationale for regulatory intervention.

Our central contribution is identifying and quantifying how liquidity requirements and redemption restrictions have fundamentally different effects on market liquidity through their impact on liquidity providers' incentives. While both interventions directly reduce mutual funds' need to liquidate assets during stress periods, they affect the supply of market liquidity in opposing ways.

Liquidity requirements mandate mutual funds to hold a minimum fraction of their portfolio in liquid assets. Our analysis shows that this directly reduces funds' need to sell illiquid assets during stress periods but partially crowds out private liquidity supply by decreasing the expected return from liquidity provision. Crucially, we demonstrate that this crowding-out effect is partial: total market liquidity increases with the regulatory requirement, improving welfare by reducing the incidence of fire sales. We characterize the optimal requirement that balances the efficiency gains from enhanced market resilience against the opportunity cost of holding lower-yielding liquid assets.

In contrast, redemption restrictions—modeled as gates that prevent withdrawals when fire-sale

discounts exceed a threshold—have counterintuitive effects on market outcomes. By eliminating precisely those states of the world where liquidity provision would be most profitable, gates substantially reduce liquidity providers' incentives to hold liquid assets. This general equilibrium response dominates the direct effect of preventing extreme fire sales, resulting in lower market liquidity and, surprisingly, a higher ex-ante probability of moderate fire sales. For most parameter values, redemption restrictions reduce welfare compared to the unregulated case.

This stark asymmetry between regulatory approaches challenges the conventional wisdom that redemption restrictions effectively mitigate liquidity risk. Our analysis suggests that policies preserving market functioning and price discovery mechanisms, even during stress periods, outperform interventions that short-circuit these market mechanisms. This insight has broad implications for financial regulation, highlighting that the optimal design must consider how interventions affect the incentives of all market participants.

We extend our model to examine whether mutual funds would build adequate liquidity buffers without regulatory requirements. The analysis reveals that competitive funds hold inefficiently low liquidity from a social welfare perspective, even when allowed to choose their liquid asset holdings endogenously. This inefficiency persists due to the same pecuniary externality identified in the baseline model, providing a theoretical foundation for mandatory liquidity requirements.

Our paper contributes to four distinct strands of literature. First, we extend research on fire sales and pecuniary externalities ([Shleifer and Vishny, 1992, 1997](#); [Lorenzoni, 2008](#); [Dávila and Korinek, 2017](#)) by analyzing how different regulatory interventions affect these externalities through their impact on endogenous market liquidity. While previous work has identified inefficiencies in markets with fire sales, our focus on comparing regulatory tools and their general equilibrium effects provides novel insights.

Second, we advance the literature on mutual fund fragility ([Chen, Goldstein and Jiang, 2010](#); [Goldstein, Jiang and Ng, 2017](#); [Feroi et al., 2014](#)) by evaluating how regulatory interventions

affect strategic complementarities in investor redemptions. These studies document that funds investing in illiquid assets are vulnerable to investor runs due to the first-mover advantage created by the liquidity mismatch between assets and liabilities. Recent evidence from the COVID-19 crisis confirms that funds holding less liquid assets experienced more severe outflows, validating our focus on liquidity-driven fragility (Jiang et al., 2022; Falato, Goldstein and Hortaçsu, 2021).

Third, we contribute to the literature on mutual fund liquidity management (Chordia, 1996; Simutin, 2014; Chernenko and Sunderam, 2016), which examines how funds balance the opportunity cost of holding cash against its benefits for managing redemption risk. Morris, Shim and Shin (2017) analyze how funds optimally manage cash buffers in response to redemption pressures. We extend this literature by modeling the general equilibrium interactions between fund liquidity management, investor redemptions, and market liquidity.

Fourth, we contribute to research on the regulation of non-bank financial intermediaries (Holmström and Tirole, 1998; Stein, 2012). Zeng (2017) demonstrates that even open-end mutual funds with flexible NAVs can experience shareholder runs if they actively manage their liquidity buffers, while Jin et al. (2022) examine how swing pricing affects mutual fund fragility. Our comparative analysis of regulatory tools and their welfare implications provides a novel framework for optimal policy design.

2 Model

This section develops a tractable model of investor redemptions in mutual funds with endogenous asset market liquidity. The framework enables us to analyze the potential market failures associated with liquidity transformation in open-ended funds and evaluate the effectiveness of various regulatory approaches.

2.1 Environment

The economy operates over three dates $t \in \{0, 1, 2\}$ with a single, homogeneous consumption good serving as the numéraire. The economy is populated by three types of risk-neutral agents: investors, mutual funds, and liquidity providers. All agents derive utility from consumption at date 2 and do not discount future consumption.

At date 0, investors allocate their endowment to mutual funds, which invest in a risky asset (for example, a portfolio of corporate bonds). Simultaneously, liquidity providers allocate their endowment between liquid and illiquid investments. After observing an aggregate shock to asset quality at date 1, investors decide whether to redeem shares from mutual funds, which then sell risky assets to liquidity providers in a competitive market to meet these redemptions. At date 2, all uncertainty is resolved, assets pay off, and agents consume.

Investors. Each investor is endowed with one unit of the consumption good at date 0 and invests it in a mutual fund. At date 1, investors gain access to a short-term investment project that transforms k units of the consumption good into $f(k)$ units at date 2. The production function $f(k)$ is increasing and concave, with $f'(0) = \infty$. To finance this investment opportunity, investors can redeem a fraction $x \in [0, 1]$ of their mutual fund shares at price s_1 .

At date 1, investors maximize expected consumption at date 2 by choosing the optimal redemption level:

$$\max_{x \in [0,1]} \{f(xs_1) + (1-x)s_2^e\} \quad (1)$$

where s_2^e is the expected share price at date 2. For an interior solution, the first-order condition yields:

$$s_1 f'(xs_1) = s_2^e \quad (2)$$

This condition balances the marginal return from the short-term investment against the expected return from continuing to hold mutual fund shares. Under our assumptions on $f(\cdot)$, redemptions

are increasing in the interim share price: $\frac{dx}{ds_1} > 0$.

Mutual funds. A representative mutual fund collects investors' endowments at date 0 and invests in a divisible risky asset with payoff at date 2 given by:

$$\tilde{y} = \begin{cases} y & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases} \quad (3)$$

The success probability π represents the asset's quality and is determined by an aggregate shock observed at date 1. From the perspective of date 0, this shock has a cumulative distribution function $H(\pi)$. Competition in the mutual fund industry ensures that funds design their redemption policies to maximize investors' expected utility.

At date 1, after observing the asset quality shock π , the mutual fund determines its share price s_1 and the quantity of the risky asset q to liquidate at market price p to maximize investors' expected consumption at date 2:

$$\max_{s_1, q} \{f(s_1 x) + (1 - x)s_2^e\}. \quad (4)$$

The maximization problem is subject to the budget constraint at date 1:

$$s_1 x = pq, \quad (5)$$

the share price at date 2, given by the pro-rata share of the remaining portfolio,

$$s_2 = \frac{(1 - q)\tilde{y}}{1 - x}, \quad (6)$$

investors' optimal redemption decision (2), and feasibility constraints $q \in [0, 1]$ and $s_1 \geq 0$.

To solve this problem, we can simplify it by using the constraints to reduce the dimension of the decision variables. First, we use the budget constraint (5) to express s_1 in terms of q . Second,

we substitute equation (6) into the objective function to replace s_2^e with the expected value of the remaining portfolio, which is $(1 - q)\pi y / (1 - x)$. Finally, we use the investors' redemption function (2) to substitute x with q .

This series of substitutions transforms the mutual fund's problem into a simpler optimization with q as the only choice variable:

$$\max_{q \in [0,1]} \{f(qp) + (1 - q)\pi y\}, \quad (7)$$

subject to

$$s_1 = \frac{pq}{x} = \frac{pq}{q^S(p; \pi)} = \frac{(1 - q)\pi y}{f'(pq)} + pq. \quad (8)$$

This reformulation separates the problem into two components: first, determining the optimal quantity q of risky assets to liquidate, which is sufficient to find the equilibrium price p ; and second, deriving the optimal share price s_1 from the pair (q, p) using equation (8).

For the first component, we focus on the maximization problem (7), which yields the first-order condition for an interior solution:

$$p f'(pq) = \pi y. \quad (9)$$

This condition equates the marginal return from investing liquidation proceeds in the short-term project with the expected return from holding the risky asset. Solving this first-order condition for q yields the interior part of the supply function:

$$q^S(p; \pi) = \frac{(f')^{-1}(\pi y / p)}{p} \quad (10)$$

where $q^S(p; \pi)$ now represents the optimal quantity of risky assets that mutual funds supply as a function of the market price p and asset quality π , deriving directly from the fund's optimization problem (7).

When the price is sufficiently high, the mutual fund optimally sells all of its risky asset holdings. This corner solution with $q = 1$ occurs when $pf'(p) \geq \pi y$. The threshold price \hat{p} at which this holds with equality is defined by $\hat{p}f'(\hat{p}) = \pi y$. Combining the interior and corner solutions allows us to characterize the mutual fund's supply function completely, expressing how the optimal quantity supplied varies with market price for any given level of asset quality:

$$q^S(p; \pi) = \begin{cases} \frac{(f')^{-1}(\pi y/p)}{p} & \text{if } p < \hat{p} \\ 1 & \text{if } p \geq \hat{p} \end{cases} \quad (11)$$

This supply function is increasing in price on the interior region, highlighting the potential feedback effects between asset prices and liquidation quantities.¹

Liquidity providers. Liquidity providers are endowed with w units of the consumption good at date 0 and face two investment opportunities. First, they can invest in a long-term project that yields $g(k)$ units of the consumption good at date 2 from k units invested at date 0. The function $g(\cdot)$ is increasing and concave with $g'(0) = \infty$. Second, they can invest in a liquid asset that yields a unit return between consecutive dates.

Crucially, only liquid assets can be used at date 1 to purchase mutual funds' risky assets. This cash-in-advance constraint creates the potential for fire sales when liquidity is scarce. At date 1, after observing the aggregate shock π , liquidity providers determine their demand for the risky asset:

At date 1, after observing the aggregate shock π , liquidity providers determine their optimal purchases of the risky asset. Given their predetermined liquidity holdings m , their demand function

¹To ensure that mutual funds' supply of the risky asset is upward-sloping in its price, we impose an additional assumption on the production function: $kf''(k) + f'(k) > 0$ for all $k > 0$.

can be expressed as:

$$q^D(p; \pi, m) = \begin{cases} \frac{m}{p} & \text{if } p < \pi y \\ \in \left[0, \frac{m}{p}\right] & \text{if } p = \pi y \\ 0 & \text{if } p > \pi y \end{cases} \quad (12)$$

This demand function reflects liquidity providers' rational behavior: when the price exceeds the fundamental value ($p > \pi y$), they don't purchase any assets; when the price equals the fundamental value ($p = \pi y$), they are indifferent about purchasing the asset; and when the price falls below the fundamental value ($p < \pi y$), they purchase as much as they can afford given their predetermined liquidity m .

At date 0, anticipating these investment opportunities, liquidity providers choose their portfolio allocation to maximize expected consumption:

$$\max_{m \in [0, w]} \{g(w - m) + m + \mathbb{E} [q^D(p; \pi, m)(\pi y - p)]\} \quad (13)$$

The first-order condition balances the marginal return of the long-term project against the unit return of the liquid asset plus the expected excess return from purchasing discounted risky assets at date 1.

3 Competitive equilibrium

This section derives the competitive equilibrium, characterizing both the risky asset market and mutual funds' optimal redemption policies. I proceed by first analyzing the equilibrium in the asset market at date 1 and then examining liquidity providers' ex-ante liquidity choice at date 0.

3.1 Asset market clearing

The equilibrium in the risky asset market is determined by the intersection of mutual funds' supply and liquidity providers' demand. To distinguish between individual and aggregate quantities, I denote the *aggregate* amount of liquid assets held by liquidity providers as M , which represents the total market liquidity available for risky asset purchases. This corresponds to the symmetric equilibrium value of the individual liquidity holding m introduced in Section 2.

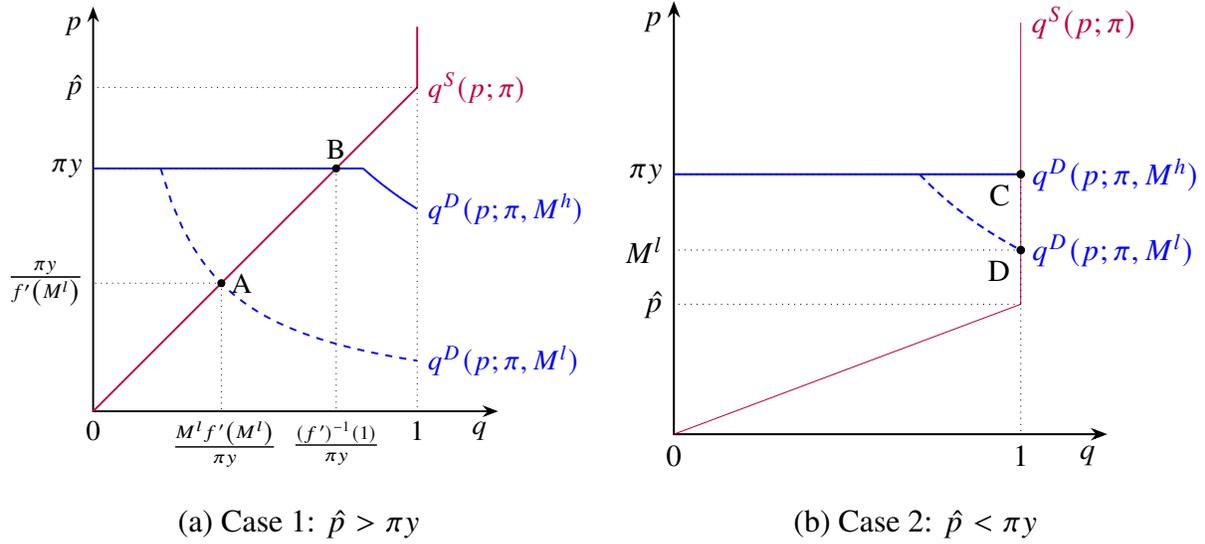


Figure 1: Demand and supply for the risky asset at date 1

This figure depicts mutual funds' supply q^S and liquidity providers' demand q^D for different levels of market liquidity M^s with $M^h > M^l$. Panel (a) shows a case where mutual funds don't sell all their assets until the price exceeds fundamental value. Panel (b) shows a case where funds may sell all their holdings at or below fundamental value.

Mutual funds' supply curve (11) has an upward-sloping part and becomes vertical when the price exceeds \hat{p} . Liquidity providers' demand curve (12) has a horizontal part at the fundamental price and a downward-sloping part for prices below fundamental value. These characteristics create four potential equilibrium configurations, labeled as points A-D in Figure 1. Each configuration corresponds to a distinct region in the parameter space of asset quality π and market liquidity M , which we refer to as equilibrium regions throughout the paper. Specifically, Region A corresponds to point A in Figure 1(a), Region B to point B in Figure 1(a), Region C to point C in Figure 1(b),

and Region D to point D in Figure 1(b).

To characterize these equilibrium regions systematically, I define a threshold on market liquidity \hat{M} , implicitly defined by $f'(\hat{M}) = 1$, such that the risky asset trades at its fundamental price regardless of the asset quality shock if $M > \hat{M}$. This follows from the fact that when $f'(M) \leq 1$, liquidity providers will never have an incentive to use all their liquidity to purchase assets, as the marginal return from the short-term investment would be lower than the unit return of simply holding the liquid asset. I also define a threshold on asset quality $\hat{\pi}$, implicitly defined by $f'(\hat{\pi}y) = 1$, such that mutual funds keep some of their risky asset holdings regardless of market liquidity if $\pi > \hat{\pi}$. This occurs because when $\pi > \hat{\pi}$, the expected return from holding the risky asset exceeds the marginal return from selling it and investing the proceeds in the short-term project, even when the price equals the fundamental value.

The four equilibrium regions reflect different combinations of constraints across market participants. In Region A, mutual funds sell some but not all of their risky assets, while liquidity providers use all their liquid funds, resulting in a fire-sale price $p = \pi y / f'(M) < \pi y$. Region B occurs when mutual funds sell some risky assets at fundamental value, with neither party constrained. In Region C, mutual funds sell all their risky assets at fundamental value. Region D represents states where mutual funds sell all their risky assets at a fire-sale price $p = M < \pi y$, but liquidity providers do not exhaust their liquidity. The derivation of the equilibrium regions is relegated to Appendix A.1.

Given these distinct regions, we can now fully characterize the equilibrium by deriving the market-clearing price $p^*(\pi; M)$ and quantity $q^*(\pi; M)$ as functions of the realized asset quality π and available market liquidity M . These equilibrium functions represent the outcomes that arise from the intersection of mutual funds' supply and liquidity providers' demand across different parameter regions, as formalized in Proposition 1.

Proposition 1. (*Asset Market Clearing*): *The price and quantity of the risky asset at date 1 are uniquely determined by the realized asset quality π and market liquidity M .*

If $M > \hat{M}$, then

$$p^*(\pi; M) = \pi y \quad \text{and} \quad q^*(\pi; M) = \begin{cases} 1 & \text{if } f'(\pi y) \geq 1 \\ \frac{(f')^{-1}(1)}{\pi y} & \text{if } f'(\pi y) < 1 \end{cases}.$$

If $M \leq \hat{M}$, then

$$p^*(\pi; M) = \begin{cases} \frac{\pi y}{f'(M)} & \text{if } \pi y > M f'(M) \\ M & \text{if } \pi y \in [M, M f'(M)] \\ \pi y & \text{if } \pi y < M \end{cases} \quad \text{and} \quad q^*(\pi; M) = \begin{cases} \frac{M f'(M)}{\pi y} & \text{if } \pi y > M f'(M) \\ 1 & \text{if } \pi y \leq M f'(M) \end{cases}$$

This proposition highlights that fire sales occur only when market liquidity is scarce ($M \leq \hat{M}$). For a fixed level of asset quality, increased market liquidity reduces the likelihood of fire sales. Conversely, for a fixed level of market liquidity, higher asset quality can paradoxically increase the likelihood of fire sales. This counterintuitive result arises because higher asset quality (larger π) increases the expected return from holding the risky asset (πy), strengthening mutual funds' incentive to retain it. Consequently, mutual funds require a higher price to voluntarily sell the asset, but when market liquidity is constrained, this higher reservation price cannot be met, leading to fire-sale conditions where $p^* < \pi y$. This demonstrates how improving fundamentals can sometimes exacerbate liquidity pressures when market liquidity is limited.

Share prices and investor redemptions. For all equilibrium regions, substituting the equilibrium price and quantity into equation (8) yields $s_1^*(\pi) = p^*(\pi; M)$, meaning the price of mutual fund shares equals the market value of the funds' risky asset holdings. This confirms that mutual funds optimally mark their shares to market, passing through fire-sale discounts to redeeming investors.

Mutual funds' budget constraint (5) determines investors' redemptions as $x^*(s_1^*(\pi)) = q^*(\pi; M)$.

This relationship illuminates a key mechanism in our model: since shares are priced at the market value of the fund's assets, the measure of redeeming shares precisely equals the quantity of risky assets sold.

3.2 Ex-ante liquidity supply

I now analyze liquidity providers' portfolio choice at date 0, which determines the aggregate market liquidity M at date 1. Having characterized how the asset market clears for any given level of liquidity, we now take a step back to understand how liquidity is optimally supplied in anticipation of these market outcomes.

To understand the incentives for liquidity provision, I define liquidity providers' payoff from holding the liquid asset at date 1 conditional on the realized π :

$$v^P(\pi; m, M) \equiv m + q^D(p^*(\pi; M); \pi, m) [\pi y - p^*(\pi; M)] \quad (14)$$

The marginal private value of liquidity at date 1 is $v^{P'}(\pi; m, M) = \partial v^P(\pi; m, M)/\partial m$. The expected marginal private value at date 0 is:

$$v_0^P(m, M) = \mathbb{E}[v^{P'}(\pi; m, M)] \quad (15)$$

The first-order condition associated with liquidity providers' optimization problem (13) can then be expressed in terms of this expected marginal private value:

$$g'(w - m^*) = v_0^P(m^*, M) \quad (16)$$

where m^* represents an individual liquidity provider's optimal choice, while M represents the aggregate market liquidity that affects equilibrium prices and quantities in the asset market.

Liquidity providers balance the foregone marginal return of investing in the long-term project with the expected marginal value of holding the liquid asset. This marginal value varies across the equilibrium regions defined in Section 3.1. In Regions B and C where the risky asset trades at its fundamental value, liquidity providers earn no surplus from purchasing the asset, and $v^{p'}(\pi; m, M) = 1$. In Region A where fire sales occur but liquidity providers use all their liquid funds, the marginal value is $v^{p'}(\pi; m, M) = f'(M)$. In Region D where fire sales occur but liquidity providers don't exhaust their liquidity, the marginal value is $v^{p'}(\pi; m, M) = \pi y/M$. The detailed derivation of these values is provided in Appendix A.2.

Proposition 2 summarizes liquidity providers' optimal investment in the liquid asset.

Proposition 2. (*Liquidity Supply*): *The investment in the liquid asset at date 0 in the competitive equilibrium satisfies:*

$$g'(w - m^*) = 1 + \mathbb{1}_{\{M \leq \hat{M}\}} \left[\underbrace{\int_{M/y}^{Mf'(M)/y} \left(\frac{\pi y}{M} - 1 \right) dH(\pi)}_{\text{Region D}} + \underbrace{\int_{Mf'(M)/y}^1 (f'(M) - 1) dH(\pi)}_{\text{Region A}} \right] \quad (17)$$

where $\mathbb{1}$ denotes the indicator function.

This equation implicitly defines a value of m^* for each M , say $m(M)$. In equilibrium, the aggregate level of liquidity M^* is the unique fixed-point of $m(M^*) = M^*$.

The equilibrium is characterized by a complementarity between individual and aggregate liquidity decisions. If aggregate liquidity is low, fire sales become more likely, creating stronger incentives for individual liquidity providers to hold liquid assets. Conversely, if aggregate liquidity is high, the expected return from purchasing discounted assets diminishes, reducing the incentive to hold liquid assets.

Proposition 3 summarizes the competitive equilibrium.

Proposition 3. (*Competitive Equilibrium*): *In the competitive equilibrium, the optimal mutual fund*

share price and investor redemptions at date 1 are given by:

$$s_1^*(\pi) = p^*(\pi; M) \quad \text{and} \quad x^*(s_1^*(\pi)) = q^*(\pi; M),$$

where $p^*(\pi; M)$ and $q^*(\pi; M)$ are given in Proposition 1, and M^* is the unique fixed-point of $m(M^*) = M^*$.

This proposition highlights that competitive mutual funds pass the market value of their risky asset portfolio to investors through their share price. When risky assets trade at fire-sale discounts, the value of mutual fund shares drops below the fundamental value of the fund's portfolio.

4 Efficiency analysis

This section analyzes the welfare implications of the competitive equilibrium by comparing it to efficient benchmarks. We first characterize the key inefficiency in the competitive equilibrium and then identify its economic source, providing the foundation for evaluating regulatory interventions in Section 5.

For the welfare analysis, we define social welfare as the expected unweighted sum of consumption across all agents at date 2:

$$\Pi(M, I(\pi)) = g(w - M) + M + \mathbb{E}[\pi y + f(I(\pi)) - I(\pi)] \quad (18)$$

where M is the aggregate investment in the liquid asset at date 0, and $I(\pi)$ is the investment in the short-term project at date 1.

4.1 Welfare benchmarks

We compare the competitive equilibrium to two relevant benchmarks:

First best. The first-best allocation maximizes social welfare subject only to resource constraints. The planner chooses the investment in the liquid asset at date 0 (M) and the investment in the short-term project at date 1 (I) to maximize equation (18).

When the economy's endowment is abundant ($w \geq \hat{w}$), the planner invests in both the liquid asset and the long-term project to equate their marginal returns:

$$f'(I^{FB}) = g'(w - M^{FB}) = 1 \quad (19)$$

When resources are scarce ($w < \hat{w}$), the planner allocates all liquid resources to the short-term project ($M^{FB} = I^{FB}$) and chooses M^{FB} to satisfy:

$$g'(w - M^{FB}) = f'(M^{FB}) > 1 \quad (20)$$

Further details on the first-best allocation are provided in Appendix ??.

Second best (constrained efficiency). We also consider a constrained planner who can only determine the investment in the liquid asset at date 0, leaving all date 1 decisions to competitive markets. This second-best approach, in the spirit of [Stiglitz \(1982\)](#), allows us to isolate inefficiencies arising from agents' ex-ante liquidity choices while respecting the constraints imposed by market mechanisms at date 1. Unlike atomistic liquidity providers, this constrained planner internalizes how the initial liquidity choice affects equilibrium prices and quantities in the asset market.

The constrained planner recognizes that the investment in the short-term project at date 1 is determined by:

$$I(\pi; M) = p^*(\pi; M)q^*(\pi; M) \quad (21)$$

where $p^*(\pi; M)$ and $q^*(\pi; M)$ are the equilibrium price and quantity from Proposition 1. This equation reflects a key market mechanism: the proceeds from mutual funds' asset sales (p^*q^*)

directly determine the resources available for investment in the short-term project, creating a link between asset market conditions and real investment efficiency.

To characterize the constrained-efficient allocation, we define the planner's payoff from holding the liquid asset at date 1 conditional on the realized π :

$$v^s(\pi; M) = M + f(p^*(\pi; M)q^*(\pi; M)) - p^*(\pi; M)q^*(\pi; M) \quad (22)$$

The marginal social value of liquidity at date 1 is $v^{s'}(\pi; M) = \partial v^s(\pi; M)/\partial M$. The expected marginal social value at date 0 is:

$$v_0^s(M) = \mathbb{E}[v^{s'}(\pi; M)] \quad (23)$$

The constrained-efficient level of liquidity M^{SB} equates this expected marginal social value with the marginal cost of providing liquidity:

$$g'(w - M^{SB}) = v_0^s(M^{SB}) \quad (24)$$

4.2 Why market liquidity is inefficiently low

We now identify the key inefficiency in the competitive equilibrium by comparing it to these welfare benchmarks.

Proposition 4. (*Inefficiency of Competitive Equilibrium*): *The competitive equilibrium features inefficiently low liquidity provision when resources are scarce:*

$$\text{If } w < \hat{w}, \text{ then } M^{CE} < M^{SB} < M^{FB}$$

$$\text{If } w \geq \hat{w}, \text{ then } M^{CE} = M^{SB} = M^{FB}$$

where M^{CE} , M^{SB} , and M^{FB} denote the supply of liquidity in the competitive equilibrium, second best, and first best, respectively.

The full proof is provided in Appendix A.4. The key insight is that when resources are scarce ($w < \hat{w}$), liquidity providers fail to internalize the full social benefit of their liquidity provision, leading to underinvestment in the liquid asset from a welfare perspective.

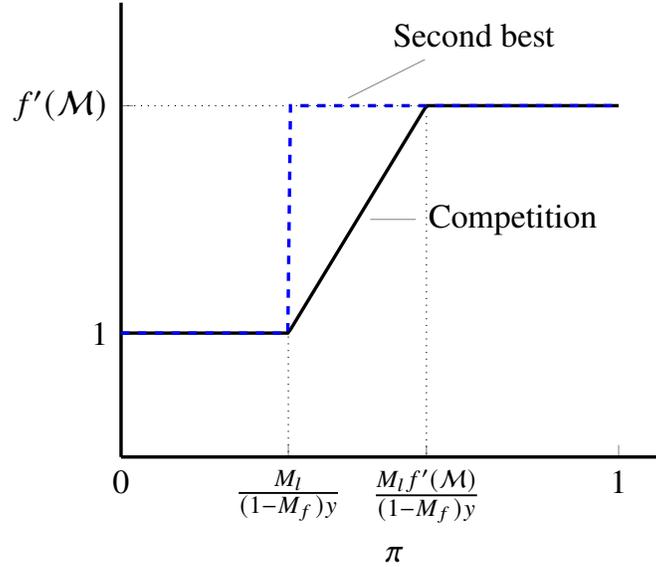


Figure 2: Marginal value of liquidity providers' investment in the liquid asset

This figure illustrates how the marginal value from the liquid asset for competitive liquidity providers (solid line) and the constrained social planner (dashed line) varies with the realized asset quality π at date 1 when the asset market is illiquid ($M < \hat{M}$).

Figure 2 provides the economic intuition behind this inefficiency by comparing the marginal private value of liquidity that guides individual decisions (solid line) with the marginal social value that would maximize welfare in the constrained-efficient benchmark (dashed line). The divergence between these values reveals why market outcomes are inefficient.

For low realizations of asset quality ($\pi < M/y$), both private and social values equal the unit return of the liquid asset, as the risky asset trades at its fundamental price. For very high realizations ($\pi > M f'(M)/y$), both values equal the marginal product of the short-term project, as all available liquidity is deployed to purchase assets during severe fire sales.

The wedge appears specifically in the intermediate range where $\pi \in [M/y, Mf'(M)/y]$. In this region, mutual funds sell all their risky assets at fire-sale prices, but the discount is not severe enough to induce liquidity providers to exhaust their liquid reserves. This creates a state-contingent wedge where the social return to liquidity exceeds its private return.

This inefficiency represents a classic pecuniary externality: liquidity providers do not internalize how their ex-ante liquidity choices affect asset prices during fire sales and, consequently, how these price effects impact investors' ability to fund productive short-term projects. This externality provides the theoretical foundation for regulatory intervention.

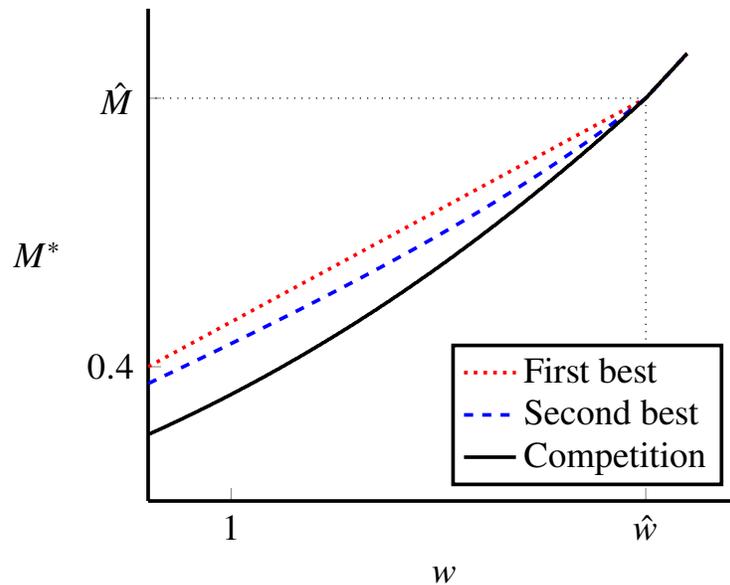


Figure 3: Inefficient liquidity provision in competitive equilibrium

This figure illustrates how the optimal liquidity supply in the first best (dotted line), second best (dashed line), and competitive equilibrium (solid line) varies with the economy's endowment w .

Figure 3 illustrates the magnitude of this inefficiency across different levels of the economy's endowment. When resources are constrained ($w < \hat{w}$), the competitive equilibrium (solid line) features systematically lower liquidity provision compared to the constrained-efficient benchmark (dashed line). This inefficiency is particularly pronounced in economies with intermediate levels of aggregate resources, where the wedge between private and social incentives is largest.

The divergence between the constrained-efficient benchmark and the first-best allocation (dotted line) captures an additional inefficiency arising from market incompleteness. Even the constrained planner cannot fully replicate the first-best allocation, which would require state-contingent transfers across agents to completely insure against the aggregate shock.

5 Regulatory interventions in mutual funds

Having established that the competitive equilibrium features inefficiently low liquidity provision due to pecuniary externalities, we now evaluate alternative regulatory approaches to address this market failure. The efficiency analysis provides three key insights that guide our evaluation: First, the competitive equilibrium is inefficient precisely because of insufficient market liquidity, especially when resources are scarce. This suggests that effective regulation should target the underprovision of liquidity directly. Second, the wedge between private and social values is state-contingent, appearing primarily in intermediate states where fire sales occur but do not deplete liquidity providers' resources. Optimal policy should be particularly attentive to addressing inefficiencies in these states. Third, the pecuniary externality operates through asset prices, which affect resource allocation. Regulatory interventions that preserve market functioning while correcting incentives are likely to outperform those that interfere with price discovery mechanisms.

We examine two regulatory interventions that have featured prominently in policy debates: liquidity requirements that operate ex-ante by mandating minimum liquid asset holdings, and redemption restrictions that function ex-post by limiting investor withdrawals during stress periods.

Before analyzing these specific regulatory tools, we first investigate whether the inefficiency we identified could be resolved through market mechanisms if mutual funds were allowed to endogenously determine their liquidity holdings. This analysis provides an important benchmark for evaluating the necessity and effectiveness of external regulatory intervention.

5.1 Endogenous mutual fund liquidity

We extend our baseline model to allow mutual funds to choose their liquid asset holdings at date 0. At this stage, a representative mutual fund determines what fraction of its portfolio to allocate to liquid versus risky assets, anticipating how this choice affects its ability to meet investor redemptions at date 1.

Specifically, the fund enters date 0 with one unit of the consumption good and chooses $m_f \in [0, 1]$ to invest in the liquid asset, with the remaining $1 - m_f$ allocated to the risky asset. At date 1, after observing the asset quality shock π , the fund's problem becomes choosing how much of its risky asset holdings to liquidate:

$$\max_{q \in [0, 1 - m_f]} \{f(qp + m_f) + (1 - m_f - q)\pi y\} \quad (25)$$

We assume that funds use their liquid asset holdings before selling risky assets to meet redemptions, consistent with empirical evidence documented in [Ma, Xiao and Zeng \(2020\)](#).

Anticipating this date 1 decision, the fund's objective at date 0 is to maximize investors' expected utility:

$$\max_{m_f \in [0, 1]} \mathbb{E} \left[f \left(q^S(p^*; \pi, m_f) p^* + m_f \right) + \left(1 - m_f - q^S(p^*; \pi, m_f) \right) \pi y \right] \quad (26)$$

where $q^S(p^*; \pi, m_f)$ is the fund's optimal asset liquidation policy at date 1 given its liquidity holdings m_f , and p^* is the equilibrium asset price. The detailed derivation of the fund's optimal liquidation policy and the resulting first-order condition for m_f are provided in [Appendix B.1](#).

Proposition 5. (*Endogenous Mutual Fund Liquidity*): *When mutual funds can endogenously choose their liquid asset holdings:*

(i) The optimal choice of liquidity m_f^* satisfies the first-order condition:

$$\mathbb{E}[\pi y] = \mathbb{E} \left[\frac{\partial \rho^p(\pi; m_f, \bar{M})}{\partial m_f} \right] \quad (27)$$

where $\rho^p(\pi; m_f, \bar{M})$ represents the fund's expected payoff from holding liquid assets.

(ii) Competitive mutual funds hold inefficiently low liquidity buffers from a social welfare perspective because the marginal social value of liquidity exceeds its private value, particularly in states where fire sales occur but liquidity providers don't exhaust their resources:

$$\frac{\partial \rho^s}{\partial M_f} - \frac{\partial \rho^p}{\partial m_f} > 0 \quad (28)$$

Figure 4 illustrates a key inefficiency in mutual funds' liquidity decisions: the marginal social value of mutual fund liquidity (dashed line) systematically exceeds its private value (solid line), particularly for intermediate realizations of the asset quality shock. The divergence is most pronounced in states where fire sales occur but liquidity providers do not exhaust their resources. This wedge arises from the same pecuniary externality identified earlier: mutual funds fail to internalize how their liquidity holdings affect asset prices and, consequently, the economy-wide efficiency of resource allocation.

The magnitude of this inefficiency varies with market conditions. In states where liquidity is abundant (low π) or severely constrained (high π), mutual funds correctly assess the value of liquidity, as shown by the overlapping lines in these regions. However, in the intermediate region where the externality is largest, there is a substantial gap between social and private valuations.

Figure 5 illustrates the equilibrium outcome when mutual funds can choose their liquidity holdings endogenously. The figure reveals a strategic substitution pattern between mutual fund liquidity and market liquidity. When liquidity providers' endowment w is small, they have a strong incentive to invest in their long-term project with high marginal returns, resulting in limited

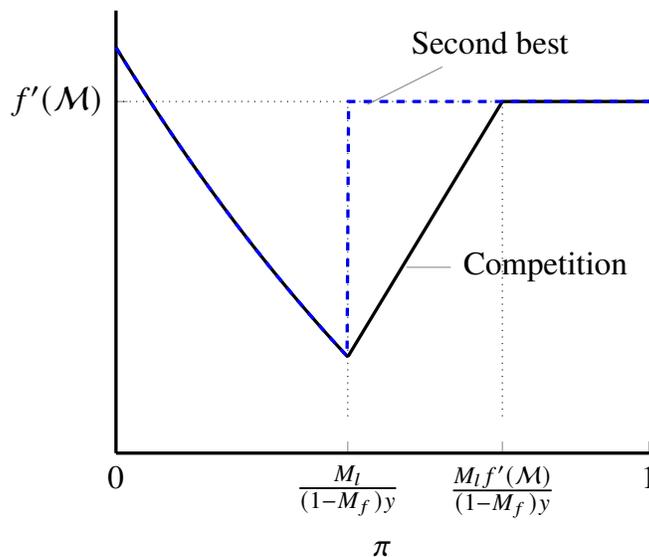


Figure 4: Marginal value of mutual fund liquidity

This figure compares the marginal private value (solid line) and social value (dashed line) of mutual fund liquidity across different realizations of the asset quality shock π . The wedge between these values is most pronounced in the intermediate region, corresponding to states where fire sales occur but liquidity providers don't exhaust their resources.

market liquidity (low M_l). Anticipating this scarcity, mutual funds respond by holding substantial liquidity buffers (high M_f). As w increases, the pattern reverses: liquidity providers supply more market liquidity (increasing M_l), allowing mutual funds to reduce their buffers (decreasing M_f) and increase investment in the higher-yielding risky asset.

This analysis establishes that allowing mutual funds to endogenously determine their liquidity holdings does not resolve the underlying inefficiency. Even with the freedom to build liquidity buffers, competitive funds hold inefficiently low liquidity from a social welfare perspective, providing a theoretical foundation for regulatory intervention. We now turn to analyzing specific regulatory approaches designed to address this inefficiency.

5.2 Liquidity requirements

Having established that mutual funds hold inefficiently low liquidity when left to their own devices, we now analyze how mandatory liquidity requirements affect market outcomes and welfare. These

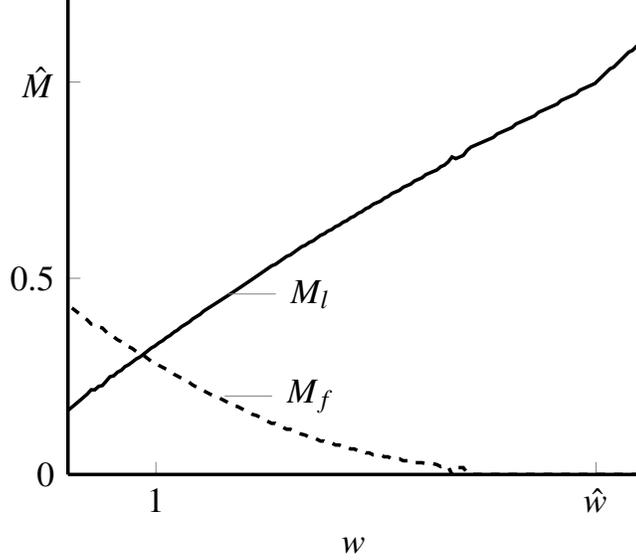


Figure 5: Equilibrium liquidity provision with endogenous mutual fund liquidity

This figure shows how liquid asset holdings by liquidity providers M_l (solid line) and mutual funds M_f (dashed line) vary with liquidity providers' endowment w . The pattern demonstrates strategic substitutability between private and mutual fund liquidity.

requirements mandate that mutual funds hold a minimum fraction m_f of their portfolio in liquid assets, directly addressing the underprovision of liquidity in the competitive equilibrium.

For analytical tractability, we assume this constraint is binding, consistent with our finding that competitive funds hold less liquidity than socially optimal. As derived in Appendix B.2, the mutual fund's optimal asset liquidation policy with a binding liquidity requirement becomes:

$$q^S(p; \pi, m_f) = \begin{cases} \frac{(f')^{-1}(\pi y/p) - m_f}{p} & \text{if } p < \hat{p} \\ 1 - m_f & \text{if } p \geq \hat{p} \end{cases} \quad (29)$$

This supply function illustrates how liquidity requirements influence resource reallocation: for any price p and asset quality π , mutual funds sell fewer risky assets when they hold more liquid assets ($m_f > 0$).

Equilibrium effects.

Proposition 6. (*Equilibrium with Liquidity Requirements*): When mutual funds are subject to a liquidity requirement $m_f \in [0, 1)$, the competitive equilibrium has the following properties:

(i) Liquidity providers reduce their holdings of the liquid asset in response to the requirement, with:

$$\frac{dM_l}{dM_f} = -\frac{v_0^{p'}(M_l + M_f)}{v_0^{p'}(M_l + M_f) + g''(w - M_l)} \in (-1, 0) \quad (30)$$

(ii) For interior values of M_f where mutual funds hold a positive amount of risky assets, total market liquidity increases with the requirement:

$$\frac{d(M_l + M_f)}{dM_f} = 1 + \frac{dM_l}{dM_f} > 0 \quad (31)$$

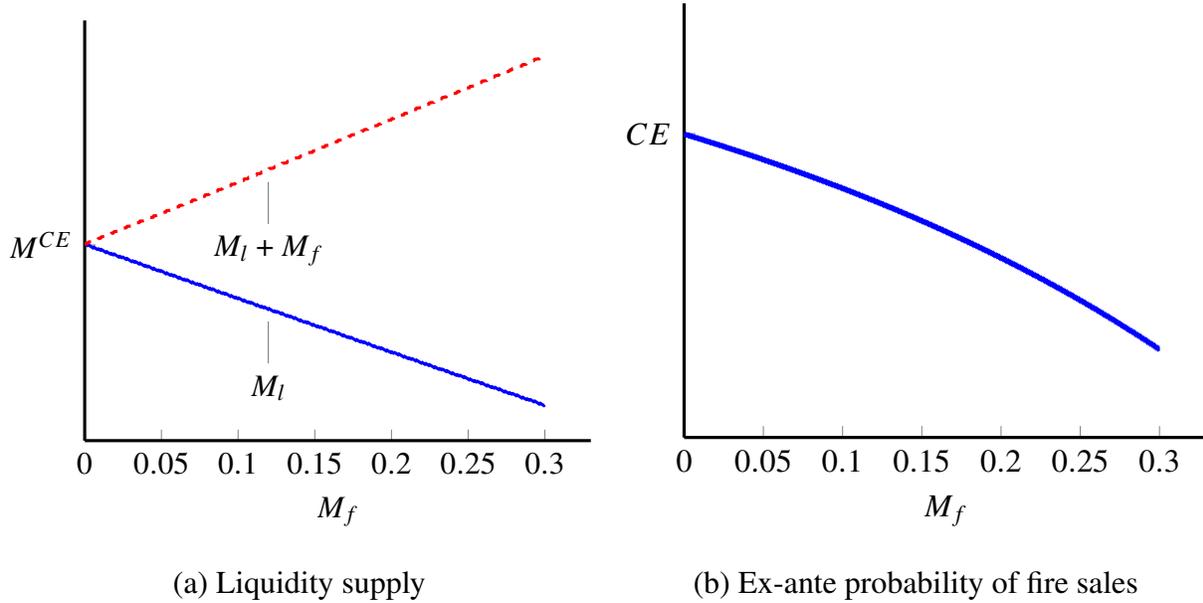


Figure 6: Equilibrium effects of liquidity requirements

Panel (a) shows the effect of the liquidity requirement M_f on liquidity providers' holdings M_l (solid line) and total market liquidity $M_l + M_f$ (dashed line). Panel (b) depicts the relationship between the liquidity requirement and the ex-ante probability of fire sales. The unregulated competitive equilibrium is represented at $M_f = 0$.

Figure 6(a) illustrates the economic trade-off described in Proposition 6. As the liquidity requirement M_f increases from zero (the unregulated equilibrium), liquidity providers reduce their holdings M_l , shown by the downward-sloping solid line. This response represents a crowding-out effect: each additional unit of mandated mutual fund liquidity reduces private liquidity provision.

Proposition 6 highlights that this crowding-out is partial: since $v_0^{p'}(\cdot) > 0$ and $g''(\cdot) < 0$, we have $-1 < \frac{dM_l}{dM_f} < 0$. This is visually confirmed by the dashed line in Figure 6(a), representing total market liquidity $M_l + M_f$, which increases with the regulatory requirement. This net increase in system-wide liquidity is a key channel through which regulation improves market outcomes.

The economic importance of this increased liquidity is evident in Figure 6(b), which shows how the ex-ante probability of fire sales initially decreases as the regulatory requirement increases. This pattern highlights how liquidity requirements can enhance market resilience by ensuring mutual funds have buffers to meet redemptions without triggering excessive asset sales.

It's important to note, however, that this relationship need not be monotonic over the entire range of possible requirements. For sufficiently large values of M_f , especially as M_f approaches 1, mutual funds would hold very little of the risky asset, dramatically reducing liquidity providers' incentive to hold liquid assets at date 0. In the extreme case where $M_f = 1$, mutual funds would hold no risky assets, eliminating the possibility of fire sales but also removing liquidity providers' motivation to supply market liquidity. The numerical example in Figure 6(b) focuses on the empirically relevant range of moderate liquidity requirements where the fire-sale reduction benefit dominates.

Welfare implications. Beyond their effect on market liquidity, we now analyze how liquidity requirements affect social welfare.

Proposition 7. (*Welfare Effects of Liquidity Requirements*):

(i) The derivative of social welfare with respect to the liquidity requirement is:

$$\begin{aligned} \frac{d\Pi}{dM_f} = & -\mathbb{E}[\pi y] + 1 + \frac{dM_l}{dM_f} [g'(w - M_l) - 1] \\ & + \mathbb{E} \left[\frac{\partial f(I(\pi; M_l, M_f))}{\partial M_f} - 1 + \frac{\partial f(I(\pi; M_l, M_f))}{\partial M_l} \frac{dM_l}{dM_f} - \frac{dM_l}{dM_f} \right] \end{aligned} \quad (32)$$

(ii) The welfare function exhibits an inverted U-shape with respect to the liquidity requirement, with an interior optimum $M_f^* \in (0, 1)$.

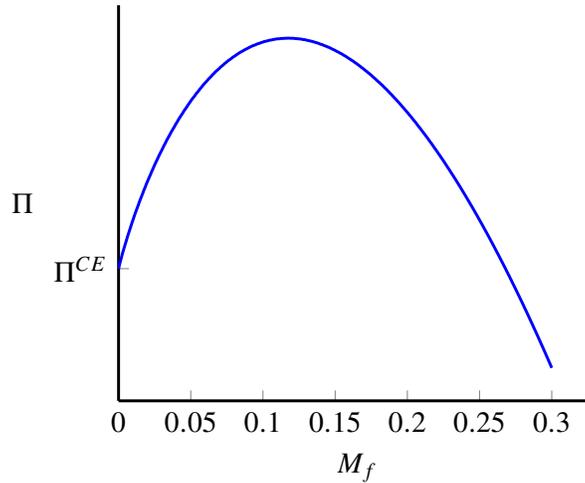


Figure 7: Social welfare under liquidity requirements

This figure illustrates the non-monotonic relationship between the liquidity requirement M_f and social welfare Π . The inverted U-shape captures the fundamental tradeoff involved in liquidity regulation. The unregulated competitive equilibrium obtains at $M_f = 0$ and features $\Pi = \Pi^{CE}$.

Figure 7 illustrates the non-monotonic relationship described in Proposition 7. The inverted U-shape captures the fundamental tradeoff at the heart of liquidity regulation. Starting from the unregulated equilibrium ($M_f = 0$), welfare initially increases as requirements rise. Beyond a certain point, the curve turns downward.

The welfare effects can be understood by examining the derivative in equation (32). The first term, $-\mathbb{E}[\pi y] + 1$, represents the direct portfolio effect: requiring mutual funds to hold the liquid asset instead of the risky asset reduces expected returns by $\mathbb{E}[\pi y] - 1$. This effect is always negative since $\mathbb{E}[\pi y] > 1$ by assumption.

The second term, $\frac{dM_l}{dM_f} [g'(w - M_l) - 1]$, captures the indirect effect through changes in liquidity providers' behavior. Since $\frac{dM_l}{dM_f} < 0$ and $g'(w - M_l) > 1$ in equilibrium, this term is also negative, reflecting the welfare cost of crowding out private liquidity provision.

The third component contains the resource allocation flexibility effect, particularly through $\mathbb{E}[\frac{\partial f(I)}{\partial M_f} - 1]$. This term is positive in states where fire sales would otherwise occur, as additional mutual fund liquidity improves the efficiency of investment in the short-term project. The remaining terms account for the indirect impact on investment efficiency through changes in liquidity providers' holdings.

At low levels of the requirement, the positive resource allocation flexibility effect dominates the negative portfolio and crowding-out effects, increasing welfare. As the requirement increases, the negative effects gradually outweigh the positive effect, leading to the inverted U-shape observed in Figure 7. The optimal liquidity requirement sets the derivative equal to zero, balancing these competing forces.

Implementation of constrained efficiency. Having established that liquidity requirements can improve welfare, we now examine whether they can implement the constrained-efficient allocation identified in Section 4.

Proposition 8. (*Implementation of Constrained Efficiency*): Let M_l^{SB} and M_f^{SB} denote the constrained-efficient levels of liquidity, with total market liquidity $M^{SB} = M_l^{SB} + M_f^{SB}$.

(i) *Liquidity requirements can implement the constrained-efficient allocation if and only if there exists $M_f^* \in [0, 1]$ such that:*

$$M_l(M_f^*) = M_l^{SB} \quad \text{and} \quad M_f^* = M_f^{SB} \quad (33)$$

where $M_l(M_f)$ denotes liquidity providers' optimal response function.

(ii) A necessary and sufficient condition for implementation is:

$$\left. \frac{dM_l}{dM_f} \right|_{(M_f^{SB}, M_l^{SB})} = \frac{v_0^{p'}(M^{SB}) - v_0^{s'}(M^{SB})}{v_0^{p'}(M^{SB})} \quad (34)$$

where $v_0^{p'}(M^{SB})$ and $v_0^{s'}(M^{SB})$ are the expected marginal private and social values of liquidity.

A central question in our analysis is whether appropriately calibrated liquidity requirements can implement the constrained-efficient allocation. As established in Section 4, the constrained-efficient allocation maximizes social welfare subject to the constraint that asset prices are determined in competitive markets at date 1. With liquidity requirements, the planner directly controls only M_f , while M_l adjusts endogenously according to liquidity providers' first-order condition (17).

The implementation challenge stems from the partial crowding-out effect: when the planner increases M_f , liquidity providers respond by reducing M_l . To achieve the constrained-efficient allocation (M_l^{SB}, M_f^{SB}) , the planner must set a requirement M_f^* such that liquidity providers optimally choose $M_l(M_f^*) = M_l^{SB}$ in response.

This requires aligning private incentives with social objectives. In the constrained-efficient allocation, the marginal social value of liquidity equals the marginal cost of providing it: $g'(w - M_l^{SB}) = v_0^{s'}(M^{SB})$. However, liquidity providers equate the marginal private value with the marginal cost: $g'(w - M_l) = v_0^{p'}(M_l + M_f)$. Since the private value typically differs from the social value due to the pecuniary externality, the competitive equilibrium diverges from the constrained optimum.

The implementation condition in equation (34) characterizes precisely how the liquidity requirement must adjust liquidity providers' incentives to bridge this gap. The left-hand side, $\frac{dM_l}{dM_f}$, measures how much private liquidity decreases when the requirement increases. The right-hand side captures the proportional wedge between the private and social values of liquidity. When these values align exactly, the crowding-out effect precisely offsets the distortion caused by the externality, implementing the constrained optimum.

In economic terms, the condition states that successful implementation requires the proportional reduction in private liquidity (due to crowding-out) to exactly match the proportional wedge between private and social values of liquidity. This is a stringent condition that represents a knife-edge case, making exact implementation challenging in practice.

The exact implementation of the constrained-efficient allocation using liquidity requirements is theoretically possible but requires the specific condition in equation (34) to hold. This condition generally will not be satisfied for arbitrary parameter values. However, our numerical simulations indicate that even when this condition is not exactly met, optimally calibrated liquidity requirements typically achieve a substantial fraction of the potential welfare gains. This makes them an effective tool for addressing mutual fund fragility in practice, even if they cannot always implement the precise constrained-efficient allocation.

5.3 Redemption restrictions

We now turn to an alternative regulatory approach that has featured prominently in policy debates: redemption restrictions. While both interventions aim to mitigate liquidity risk, they operate through fundamentally different mechanisms, as our analysis will reveal.

We analyze redemption restrictions, modeled as gates that prevent investor withdrawals when the fire-sale discount exceeds a threshold $\delta \in [0, 1]$. Formally, the policy imposes a price floor:

$$p \geq (1 - \delta)\pi y \tag{35}$$

This price floor representation captures the economic essence of redemption gates: by restricting redemptions when asset prices fall below a certain threshold relative to fundamental value, gates effectively prevent the price from dropping below that threshold. The parameter δ represents the maximum permitted price discount relative to fundamental value. Small values of δ correspond

to strict gates that activate even for mild fire sales, while large values represent looser policies that only bind during severe market stress.

When this constraint binds, it directly limits funds' ability to reallocate resources by capping the quantity of assets that mutual funds can sell. This constraint on resource reallocation represents a fundamental departure from the flexibility preserved under liquidity requirements.

Equilibrium effects.

Proposition 9. (*Equilibrium with Redemption Gates*): *When mutual funds are subject to redemption gates with threshold $\delta \in [0, 1]$, the competitive equilibrium has the following properties:*

(i) *Liquidity providers reduce their holdings as gates become more restrictive (lower δ):*

$$\frac{dM_l}{d\delta} > 0 \quad (36)$$

(ii) *When the unregulated price would violate the price floor constraint, the equilibrium price and quantity are:*

$$p^* = (1 - \delta)\pi y \quad \text{and} \quad q^* = \min \left\{ \frac{M_l}{(1 - \delta)\pi y}, 1 \right\} \quad (37)$$

The general equilibrium effect of redemption gates differs significantly from that of liquidity requirements. Figure 8(a) illustrates the relationship described in Proposition 9. For values of δ close to 1, the price floor $(1 - \delta)\pi y$ never binds in equilibrium for any realization of the asset quality shock π , leaving liquidity provision unchanged from the competitive equilibrium.

As δ decreases, gates begin to affect equilibrium outcomes through two channels. First, for realizations of π where the unregulated price would fall below $(1 - \delta)\pi y$, gates directly prevent extreme fire sales by enforcing the price floor in equation (37). Second, and more importantly, gates alter liquidity providers' ex-ante incentives by modifying their expected returns from liquidity provision.

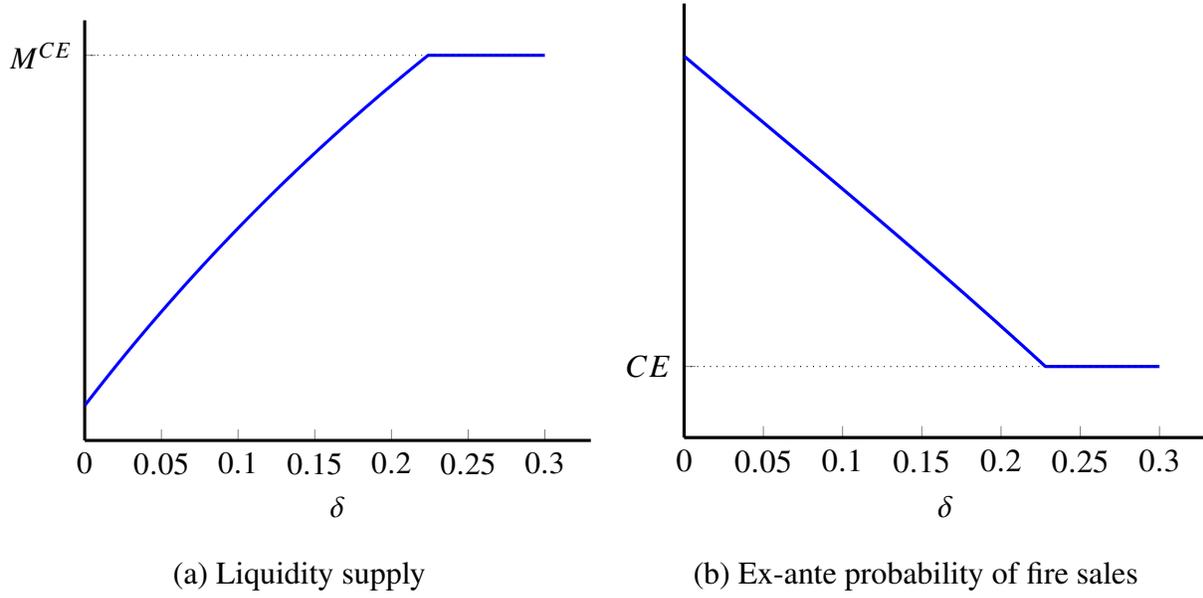


Figure 8: Equilibrium effects of redemption gates

Panel (a) illustrates how redemption gates affect equilibrium liquidity supply M_l . Panel (b) shows the relationship between the gate threshold δ and the ex-ante probability of fire sales. The unregulated equilibrium corresponds to $\delta = 1$.

When gates bind, the marginal private value of liquidity becomes:

$$\frac{\partial v^P(\pi; m, M)}{\partial m} = \frac{\delta}{1 - \delta} \quad (38)$$

Since this value decreases with δ , stricter gates (lower δ) reduce the expected return from holding liquidity, causing liquidity providers to decrease their liquid holdings as indicated by equation (36).

This reduction in market liquidity has important consequences for the probability of fire sales, as shown in Figure 8(b). While gates directly prevent extreme fire sales when they bind, they indirectly increase the ex-ante probability of moderate fire sales by reducing market liquidity. With less liquidity available, the market becomes more susceptible to fire sales in states where gates remain inactive. This illustrates a fundamental trade-off of ex-post intervention policies: they may eliminate tail events but increase the likelihood of moderate stress events through their effect on ex-ante incentives.

Welfare implications. Having characterized the equilibrium effects of gates, we now analyze their welfare implications, which reveal important differences from liquidity requirements.

Proposition 10. (*Welfare Effects of Redemption Gates*):

(i) *The derivative of social welfare with respect to the gate threshold δ can be decomposed as:*

$$\frac{d\Pi}{d\delta} = \underbrace{\frac{dM_l}{d\delta} \left[g'(w - M_l) - 1 + \mathbb{E} \left[\frac{\partial f(I)}{\partial M_l} - 1 \right] \right]}_{\text{indirect effect via market liquidity}} + \underbrace{\mathbb{E} \left[\frac{\partial f(I)}{\partial \delta} - \frac{\partial I}{\partial \delta} \right]}_{\text{direct effect on resource allocation}} \quad (39)$$

(ii) *The welfare function is non-monotonic in the gate threshold δ , with a potential interior optimum $\delta^* \in (0, 1)$.*

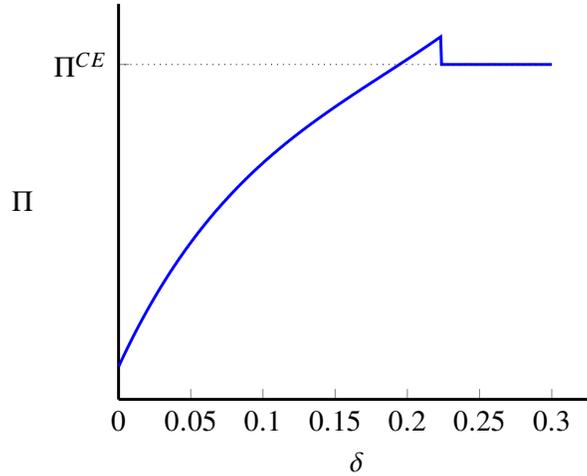


Figure 9: Welfare under redemption gates

This figure illustrates how social welfare varies with the threshold on market distress δ governing the activation of redemption gates. The non-monotonic relationship reveals the competing economic forces at play. Π^{CE} denotes the unregulated benchmark.

Figure 9 illustrates the relationship between the gate threshold δ and social welfare, reflecting the interplay between the direct and indirect effects described in equation (39). The welfare implications of redemption gates can be understood by examining each component of this derivative.

The first term represents the indirect effect through changes in market liquidity. Since $\frac{dM_l}{d\delta} > 0$

(liquidity decreases as gates become tighter) and $g'(w - M_l) > 1$ in equilibrium, this term is positive—reduced liquidity provision lowers welfare. This effect operates through the general equilibrium response of liquidity providers, who reduce their liquid holdings as gates become more restrictive, resulting in less market support for asset prices.

The second term captures the direct effect on resource allocation efficiency at date 1. This term reflects two competing forces. On one hand, gates directly improve resource allocation efficiency in states with severe fire sales by preventing excessive discounts that distort investment decisions. When fire-sale prices would otherwise be far below fundamental value, the marginal product of the short-term project $f'(I)$ significantly exceeds the expected return from the long-term asset πy , creating a large welfare loss. Gates mitigate this distortion by establishing a floor on prices. On the other hand, gates constrain resource reallocation by limiting the quantity of assets that mutual funds can sell. This constraint reduces welfare in states where some reallocation would be efficient.

The non-monotonic pattern in Figure 9 emerges from these competing effects. For high values of δ (close to 1), gates remain inactive for most realizations of π , so both effects are negligible and welfare equals the competitive benchmark. As δ decreases to an intermediate range, the direct effect may dominate in states with extreme fire sales, potentially leading to welfare improvements. As δ decreases further, the negative indirect effect through reduced market liquidity typically dominates, causing welfare to decrease below the unregulated benchmark.

These results suggest that while carefully calibrated gates might improve welfare in specific cases by preventing severely distorted asset prices, they generally reduce welfare due to their adverse effect on market liquidity. This highlights the importance of accounting for the general equilibrium effects of regulatory interventions, particularly their impact on private incentives for liquidity provision.

Implementation limitations. Unlike liquidity requirements, redemption gates face fundamental limitations in implementing the constrained-efficient allocation, as the following proposition

establishes.

Proposition 11. (*Implementation Limitations of Redemption Gates*):

(i) For gates to implement the constrained-efficient allocation, there must exist a threshold $\delta^* \in [0, 1]$ such that:

$$M_l(\delta^*) = M_l^{SB} \quad \text{and} \quad I(\pi; M_l(\delta^*), \delta^*) = I^{SB}(\pi) \quad \forall \pi \quad (40)$$

where $I^{SB}(\pi)$ is the constrained-efficient investment function.

(ii) This requires the following condition to hold for all π where gates bind:

$$(1 - \delta^*)\pi y \cdot q^{Gate}(\pi; \delta^*) = I^{SB}(\pi) \quad (41)$$

(iii) The investment in the short-term project under gates is:

$$I(\pi; M_l, \delta) = \begin{cases} \min\{p^*(\pi; M_l)q^*(\pi; M_l), M_l\} & \text{if } p^*(\pi; M_l) \geq (1 - \delta)\pi y \\ (1 - \delta)\pi y \cdot q^{Gate}(\pi; \delta) & \text{if } p^*(\pi; M_l) < (1 - \delta)\pi y \end{cases} \quad (42)$$

Redemption gates face fundamental limitations in implementing the constrained-efficient allocation. The implementation requires not only matching the constrained-efficient level of market liquidity but also achieving the efficient investment in the short-term project across all possible realizations of the asset quality shock.

The key challenge arises from equation (41), which must hold for all states where gates bind. Since the constrained-efficient investment function $I^{SB}(\pi)$ varies continuously with π , while gates impose a discrete constraint on resource reallocation that depends only on the gate threshold δ , this equation cannot generally be satisfied for all relevant states with a single gate threshold.

This implementation challenge stems from the rigid nature of redemption gates. Unlike liquidity requirements, which preserve market functioning while enhancing system-wide resilience, gates directly interfere with the price mechanism, preventing it from performing its allocative function. The constraint on resource reallocation becomes particularly costly in states where some reallocation would be efficient but gates prevent it.

Additionally, gates face significant calibration challenges. The optimal threshold depends on the distribution of asset quality shocks and the relative importance of preventing extreme fire sales versus maintaining liquidity providers' incentives. A threshold that is too loose may fail to prevent extreme fire sales, while a threshold that is too tight may severely reduce market liquidity and constrain efficient resource reallocation.

These limitations explain why gates generally cannot implement the constrained-efficient allocation and are typically inferior to liquidity requirements as a regulatory approach. While they can theoretically improve welfare in specific cases by preventing severely distorted asset prices, their practical implementation faces substantial challenges, and they often reduce welfare relative to the unregulated equilibrium due to their adverse effect on market liquidity.

6 Conclusion

This paper develops an equilibrium framework to evaluate regulatory interventions addressing liquidity risk in open-ended mutual funds. Our analysis yields three key insights. First, the competitive equilibrium features inefficiently low market liquidity due to a pecuniary externality: liquidity providers fail to internalize how their portfolio choices affect asset prices during fire sales and, consequently, the efficiency of resource allocation.

Second, liquidity requirements and redemption restrictions have fundamentally different effects on market outcomes. Liquidity requirements operate ex-ante by increasing system-wide resilience, despite partially crowding out private liquidity provision. In contrast, redemption restrictions

operate ex-post by constraining resource reallocation, which substantially reduces incentives for private liquidity provision by eliminating states with the highest returns to liquidity.

Third, these regulatory approaches have asymmetric welfare implications. Liquidity requirements improve welfare by enhancing market resilience while preserving price discovery mechanisms. Redemption restrictions generally reduce welfare, as their adverse effect on ex-ante liquidity provision typically dominates their direct benefit of preventing extreme fire sales.

Our analysis challenges the conventional view that redemption restrictions effectively mitigate liquidity risk in mutual funds. The optimal regulatory design must consider how interventions affect market functioning through their impact on the incentives of all market participants. More broadly, our findings highlight that the efficacy of financial regulation depends crucially on its general equilibrium effects, not just on the resilience of directly regulated institutions.

The model can be extended to incorporate information asymmetries, fund heterogeneity, strategic investor interactions, and alternative sources of market failure. Future empirical work could test the model's predictions regarding the differential effects of regulatory approaches on market liquidity and fund behavior.

References

- Allen, Franklin, and Douglas Gale.** 1994. "Limited market participation and volatility of asset prices." *The American Economic Review*, 933–955.
- Allen, Franklin, and Douglas Gale.** 2005. "From cash-in-the-market pricing to financial fragility." *Journal of the European Economic Association*, 535–546.
- Chen, Qi, Itay Goldstein, and Wei Jiang.** 2010. "Payoff complementarities and financial fragility: Evidence from mutual fund outflows." *Journal of Financial Economics*, 97(2): 239–262.

- Chernenko, Sergey, and Adi Sunderam.** 2016. “Liquidity transformation in asset management: Evidence from the cash holdings of mutual funds.” National Bureau of Economic Research.
- Chordia, Tarun.** 1996. “The structure of mutual fund charges.” *Journal of financial Economics*, 41(1): 3–39.
- Dávila, Eduardo, and Anton Korinek.** 2017. “Pecuniary Externalities in Economies with Financial Frictions.” *The Review of Economic Studies*.
- Falato, Antonio, Itay Goldstein, and Ali Hortaçsu.** 2021. “Financial fragility in the COVID-19 crisis: The case of investment funds in corporate bond markets.” *Journal of Monetary Economics*, 123: 35–52.
- Federal Reserve Board.** 2020. “Financial Stability Report.” November.
- Feroli, Michael, Anil K Kashyap, Kermit L Schoenholtz, and Hyun Song Shin.** 2014. “Market tantrums and monetary policy.”
- Goldstein, Itay, Hao Jiang, and David T Ng.** 2017. “Investor flows and fragility in corporate bond funds.” *Journal of Financial Economics*, 126(3): 592–613.
- Haddad, Valentin, Alan Moreira, and Tyler Muir.** 2021. “When selling becomes viral: Disruptions in debt markets in the COVID-19 crisis and the Fed’s response.” *The Review of Financial Studies*, 34(11): 5309–5351.
- Holmström, Bengt, and Jean Tirole.** 1998. “Private and public supply of liquidity.” *Journal of Political Economy*, 106(1): 1–40.
- Jiang, Hao, Yi Li, Zheng Sun, and Ashley Wang.** 2022. “Does mutual fund illiquidity introduce fragility into asset prices? Evidence from the corporate bond market.” *Journal of Financial Economics*, 143(1): 277–302.
- Jin, Dunhong, Marcin Kacperczyk, Bige Kahraman, and Felix Suntheim.** 2022. “Swing pricing and fragility in open-end mutual funds.” *The Review of Financial Studies*, 35(1): 1–50.

- Kargar, Mahyar, Benjamin Lester, David Lindsay, Shuo Liu, Pierre-Olivier Weill, and Diego Zúñiga.** 2021. “Corporate bond liquidity during the COVID-19 crisis.” *The Review of Financial Studies*, 34(11): 5352–5401.
- Lorenzoni, Guido.** 2008. “Inefficient credit booms.” *The Review of Economic Studies*, 75(3): 809–833.
- Ma, Yiming, Kairong Xiao, and Yao Zeng.** 2020. “Mutual fund liquidity transformation and reverse flight to liquidity.” *Available at SSRN 3640861*.
- Morris, Stephen, Ilhyock Shim, and Hyun Song Shin.** 2017. “Redemption risk and cash hoarding by asset managers.” *Journal of Monetary Economics*, 89: 71–87.
- Shleifer, Andrei, and Robert W Vishny.** 1992. “Liquidation values and debt capacity: A market equilibrium approach.” *The Journal of Finance*, 47(4): 1343–1366.
- Shleifer, Andrei, and Robert W Vishny.** 1997. “The limits of arbitrage.” *The Journal of Finance*, 52(1): 35–55.
- Simutin, Mikhail.** 2014. “Cash holdings and mutual fund performance.” *Review of Finance*, 18(4): 1425–1464.
- Stein, Jeremy C.** 2012. “Monetary policy as financial stability regulation.” *The Quarterly Journal of Economics*, 127(1): 57–95.
- Stiglitz, Joseph E.** 1982. “The inefficiency of the stock market equilibrium.” *The Review of Economic Studies*, 49(2): 241–261.
- Zeng, Yao.** 2017. “A Dynamic Theory of Mutual Fund Runs and Liquidity Management.”

Appendix

A Competitive Equilibrium and Efficiency: Technical Derivations

A.1 Derivation of equilibrium regions

This section provides detailed derivations for the equilibrium regions described in Section 3.1.

Region A: Interior solution with binding liquidity constraint. At Point A in Figure 1(a), liquidity providers' resource constraint binds ($pq = M$). Substituting this into mutual funds' first-order condition (9) yields:

$$p^*(\pi; M) = \frac{\pi y}{f'(M)} \quad \text{and} \quad q^*(\pi; M) = \frac{M f'(M)}{\pi y} \quad (\text{A.1})$$

For a valid equilibrium, we require: (i) $q^* < 1$, which implies $M f'(M) < \pi y$, and (ii) $p^* < \pi y$, which implies $1 < f'(M)$ or equivalently $M < \hat{M}$, where \hat{M} is defined by $f'(\hat{M}) = 1$.

Region B: Interior solution with non-binding liquidity constraint. At Point B in Figure 1(a), the asset trades at its fundamental value:

$$p^*(\pi; M) = \pi y \quad \text{and} \quad q^*(\pi; M) = \frac{(f')^{-1}(1)}{\pi y} \quad (\text{A.2})$$

For this equilibrium, we require: (i) $q^* < 1$, implying $(f')^{-1}(1) < \pi y$ or equivalently $\pi > \hat{\pi}$, where $\hat{\pi}$ is defined by $f'(\hat{\pi}y) = 1$, and (ii) $p^* q^* < M$, implying $(f')^{-1}(1) < M$ or equivalently $M > \hat{M}$.

Region C: Corner solution with non-binding liquidity constraint. At Point C in Figure 1(b), mutual funds sell all assets at fundamental value:

$$p^*(\pi; M) = \pi y \quad \text{and} \quad q^*(\pi; M) = 1 \quad (\text{A.3})$$

This equilibrium requires: (i) $pf'(p) \geq \pi y$ at $p = \pi y$, which simplifies to $f'(\pi y) \geq 1$ or $\pi \leq \hat{\pi}$, and (ii) $p^*q^* < M$, implying $\pi y < M$.

Region D: Corner solution with binding liquidity constraint. At Point D in Figure 1(b), mutual funds sell all assets at a fire-sale price:

$$p^*(\pi; M) = M \quad \text{and} \quad q^*(\pi; M) = 1 \quad (\text{A.4})$$

For this equilibrium, we require: (i) $pf'(p) \geq \pi y$ at $p = M$, implying $Mf'(M) \geq \pi y$, and (ii) $p^* < \pi y$, implying $M < \pi y$.

A.2 Private value of liquidity

This section derives the marginal private value of liquidity $v^{p'}(\pi; m, M) = \partial v^p(\pi; m, M)/\partial m$ across equilibrium regions, where $v^p(\pi; m, M)$ is defined in equation (14).

Region A: With $p^*(\pi; M) = \pi y/f'(M)$ and $q^D(p^*; \pi, m) = m/p^*$, we have:

$$v^p(\pi; m, M) = m + \frac{m}{p^*} [\pi y - p^*] = m + mf'(M) - m = mf'(M) \quad (\text{A.5})$$

Therefore, $v^{p'}(\pi; m, M) = f'(M)$.

Regions B and C: Since the asset trades at fundamental value ($p^*(\pi; M) = \pi y$), liquidity providers earn no excess return:

$$v^p(\pi; m, M) = m + q^D(p^*; \pi, m) \times 0 = m \quad (\text{A.6})$$

Thus, $v^{p'}(\pi; m, M) = 1$.

Region D: With $p^*(\pi; M) = M$ and $q^D(p^*; \pi, m) = 1$:

$$v^P(\pi; m, M) = m + \pi y - M \quad (\text{A.7})$$

In a symmetric equilibrium where $M = m$, the marginal private value is:

$$v^{P'}(\pi; m, M) = 1 + \frac{\partial}{\partial m}(\pi y - m) = \frac{\pi y}{M} \quad (\text{A.8})$$

Combining these results:

$$v^{P'}(\pi; m, M) = \begin{cases} f'(M) & \text{if in Region A} \\ 1 & \text{if in Regions B or C} \\ \frac{\pi y}{M} & \text{if in Region D} \end{cases} \quad (\text{A.9})$$

A.3 Social value of liquidity

This section derives the marginal social value of liquidity $v^{s'}(\pi; M) = \partial v^s(\pi; M)/\partial M$, where $v^s(\pi; M)$ is defined in equation (22).

Regions A and D: In both regions, $p^*(\pi; M)q^*(\pi; M) = M$, which yields:

$$v^s(\pi; M) = M + f(M) - M = f(M) \quad (\text{A.10})$$

Therefore, $v^{s'}(\pi; M) = f'(M)$.

Regions B and C: Since $p^*(\pi; M)q^*(\pi; M)$ is independent of M in these regions:

$$\frac{\partial}{\partial M}[f(p^*q^*) - p^*q^*] = 0 \quad (\text{A.11})$$

Thus, $v^{s'}(\pi; M) = 1$.

Combining these results:

$$v^{s'}(\pi; M) = \begin{cases} f'(M) & \text{if in Regions A or D} \\ 1 & \text{if in Regions B or C} \end{cases} \quad (\text{A.12})$$

A.4 Proof of Proposition 4

The proof proceeds in two steps:

Step 1: For $w \geq \hat{w}$, we show that $M^{CE} = M^{SB} = M^{FB}$.

When $w \geq \hat{w}$, the endowment is sufficient to establish $M > \hat{M}$ in equilibrium. In this case, the risky asset always trades at its fundamental value, and the first-order conditions for liquidity provision are identical across all three regimes:

$$g'(w - M) = 1 \quad (\text{A.13})$$

This yields $M^{CE} = M^{SB} = M^{FB} = g'^{-1}(1)$, establishing efficiency.

Step 2: For $w < \hat{w}$, we show that $M^{CE} < M^{SB} < M^{FB}$.

The comparison between M^{SB} and M^{FB} follows from the definitions. The first-best allocation internalizes all effects and has complete freedom in allocating resources, while the second-best allocation is constrained by market mechanisms at date 1.

To compare M^{CE} and M^{SB} , we examine the first-order conditions (17) and (24). The left-hand sides are identical, so any difference must arise from differing marginal values of liquidity.

From Proposition 2 and our derivation of the marginal social value of liquidity in Section 4, the

difference between the marginal social and private values of liquidity at date 1 is:

$$v^{s'}(\pi; M) - v^{p'}(\pi; m, M) = \begin{cases} f'(M) - \frac{\pi y}{M} > 0 & \text{if } \pi y \in [M, Mf'(M)] \\ 0 & \text{otherwise} \end{cases} \quad (\text{A.14})$$

The inequality $f'(M) - \frac{\pi y}{M} > 0$ holds in Region D because $\pi y < Mf'(M)$ implies $\frac{\pi y}{M} < f'(M)$.

Since the marginal social value of liquidity strictly exceeds the marginal private value in Region D, and they are equal elsewhere, the expected marginal social value exceeds the expected marginal private value:

$$v_0^s(M) > v_0^p(M) \quad (\text{A.15})$$

Given the identical left-hand sides in (17) and (24), this implies $M^{CE} < M^{SB}$, establishing the result.

Note that the region where the marginal values differ corresponds exactly to Region D in Figure 1(b), where mutual funds sell all their risky assets at fire-sale prices, but the discount is not severe enough to induce liquidity providers to use all their liquid funds.

B Regulatory Interventions: Technical Derivations

B.1 Endogenous mutual fund liquidity

B.1.1 Competitive equilibrium with endogenous mutual fund liquidity

At date 1, after observing the asset quality shock π , the fund's problem becomes choosing how much of its risky asset holdings to liquidate:

$$\max_{q \in [0, 1-m_f]} \{f(qp + m_f) + (1 - m_f - q)\pi y\} \quad (\text{B.16})$$

The first-order condition for an interior solution is:

$$p f'(qp + m_f) = \pi y \quad (\text{B.17})$$

Solving for q yields the supply function:

$$q^S(p; \pi, m_f) = \frac{(f')^{-1}(\pi y/p) - m_f}{p} \quad (\text{B.18})$$

A corner solution with $q = 1 - m_f$ occurs when $p f'((1 - m_f)p + m_f) \geq \pi y$.

Given this supply function and liquidity providers' demand function (12), the equilibrium price and quantity in the asset market are determined by market clearing conditions. The equilibrium conditions yield four possible configurations, corresponding to the regions identified in Proposition 1, but now accounting for mutual funds' initial liquidity holdings m_f :

Region A: When mutual funds sell some but not all of their risky assets and liquidity providers use all their liquid funds:

$$p^*(\pi; M_l, M_f) = \frac{\pi y}{f'(M_l + M_f)} \quad \text{and} \quad q^*(\pi; M_l, M_f) = \frac{M_l f'(M_l + M_f)}{\pi y} - \frac{M_f f'(M_l + M_f)}{p^*} \quad (\text{B.19})$$

Simplifying the expression for q^* :

$$q^*(\pi; M_l, M_f) = \frac{M_l f'(M_l + M_f)}{\pi y} - \frac{M_f f'(M_l + M_f)}{\pi y / f'(M_l + M_f)} = \frac{M_l f'(M_l + M_f) - M_f [f'(M_l + M_f)]^2}{\pi y} \quad (\text{B.20})$$

In this region, the marginal private value of mutual fund liquidity is:

$$\frac{\partial \rho^p}{\partial m_f} = f'(M_l + M_f) \quad (\text{B.21})$$

This follows from the envelope theorem: the marginal value equals the marginal product of the

short-term investment when evaluated at the total liquidity $M_l + M_f$.

Region B: When mutual funds sell some but not all of their risky assets and the price equals the fundamental value:

$$p^*(\pi; M_l, M_f) = \pi y \quad \text{and} \quad q^*(\pi; M_l, M_f) = \frac{(f')^{-1}(1) - M_f}{\pi y} \quad (\text{B.22})$$

In this region, the marginal private value is:

$$\frac{\partial \rho^p}{\partial m_f} = 1 \quad (\text{B.23})$$

Since the asset trades at its fundamental value, the marginal value of mutual fund liquidity equals the unit return of the liquid asset.

Region C: When mutual funds sell all their risky assets and the price equals the fundamental value:

$$p^*(\pi; M_l, M_f) = \pi y \quad \text{and} \quad q^*(\pi; M_l, M_f) = 1 - M_f \quad (\text{B.24})$$

In this region:

$$\frac{\partial \rho^p}{\partial m_f} = f'((1 - m_f)\pi y + m_f) \quad (\text{B.25})$$

This follows because additional mutual fund liquidity replaces liquidation proceeds from selling the risky asset, yielding the marginal product evaluated at the total investment in the short-term project.

Region D: When mutual funds sell all their risky assets and liquidity providers use some but not all of their liquid funds:

$$p^*(\pi; M_l, M_f) = \frac{M_l}{1 - M_f} \quad \text{and} \quad q^*(\pi; M_l, M_f) = 1 - M_f \quad (\text{B.26})$$

In this region:

$$\frac{\partial \rho^p}{\partial m_f} = f' \left(\frac{(1 - m_f)M_l}{1 - M_f} + m_f \right) \quad (\text{B.27})$$

To derive this expression, note that the total investment in the short-term project is:

$$I = p^* \cdot q^* + m_f = \frac{M_l}{1 - M_f} \cdot (1 - M_f) + m_f = M_l + m_f \quad (\text{B.28})$$

However, as m_f changes, both p^* and q^* adjust. The marginal effect on investment is:

$$\frac{\partial I}{\partial m_f} = \frac{\partial(p^* \cdot q^*)}{\partial m_f} + 1 = \frac{\partial}{\partial m_f} \left(\frac{M_l}{1 - M_f} \cdot (1 - M_f) \right) + 1 = \frac{\partial M_l}{\partial m_f} + 1 \quad (\text{B.29})$$

Given the strategic substitution effect, $\frac{\partial M_l}{\partial m_f} < 0$, which affects the marginal value accordingly.

Returning to the date 0 problem, the first-order condition that characterizes the optimal choice of m_f is:

$$\mathbb{E}[\pi y] = \mathbb{E} \left[\frac{\partial \rho^p(\pi; m_f, \bar{M})}{\partial m_f} \right] \quad (\text{B.30})$$

where $\rho^p(\pi; m_f, \bar{M}) = f(q^S(p^*; \pi, m_f)p^* + m_f) + (1 - m_f - q^S(p^*; \pi, m_f))\pi y$ represents the fund's expected payoff from holding liquid assets, and $\bar{M} = (M_f, M_l)$ denotes the aggregate market liquidity.

In equilibrium, the mutual fund's liquidity choice m_f^* must satisfy the above condition with $M_f^* = m_f^*$ (fixed point) and M_l^* being the equilibrium liquidity supplied by liquidity providers.

To derive the strategic substitution pattern between mutual fund liquidity M_f and market liquidity M_l illustrated in Figure 5, we totally differentiate the system of first-order conditions with respect to liquidity providers' endowment w :

For mutual funds:

$$\frac{d}{dw} \mathbb{E} \left[\frac{\partial \rho^p(\pi; M_f, \bar{M})}{\partial m_f} \right] = 0 \quad (\text{B.31})$$

For liquidity providers:

$$g''(w - M_l) \cdot \left(1 - \frac{dM_l}{dw}\right) + \frac{d}{dw} \mathbb{E} \left[\frac{\partial v^p(\pi; M_l, \bar{M})}{\partial m} \right] = 0 \quad (\text{B.32})$$

The term $\frac{d}{dw} \mathbb{E} \left[\frac{\partial \rho^p(\pi; M_f, \bar{M})}{\partial m_f} \right]$ can be expanded using the chain rule:

$$\frac{d}{dw} \mathbb{E} \left[\frac{\partial \rho^p(\pi; M_f, \bar{M})}{\partial m_f} \right] = \mathbb{E} \left[\frac{\partial^2 \rho^p}{\partial m_f^2} \cdot \frac{dM_f}{dw} + \frac{\partial^2 \rho^p}{\partial m_f \partial M_l} \cdot \frac{dM_l}{dw} \right] \quad (\text{B.33})$$

Setting this equal to zero and solving for $\frac{dM_f}{dw}$:

$$\frac{dM_f}{dw} = - \frac{\mathbb{E} \left[\frac{\partial^2 \rho^p}{\partial m_f \partial M_l} \right]}{\mathbb{E} \left[\frac{\partial^2 \rho^p}{\partial m_f^2} \right]} \cdot \frac{dM_l}{dw} \quad (\text{B.34})$$

Similarly, for liquidity providers:

$$\frac{dM_l}{dw} = \frac{g''(w - M_l) - \mathbb{E} \left[\frac{\partial^2 v^p}{\partial m \partial M_f} \right] \cdot \frac{dM_f}{dw}}{g''(w - M_l) + \mathbb{E} \left[\frac{\partial^2 v^p}{\partial m^2} \right]} \quad (\text{B.35})$$

Solving this system of equations yields expressions for $\frac{dM_f}{dw}$ and $\frac{dM_l}{dw}$. Under our assumptions on the production functions, we can show that $\frac{dM_l}{dw} > 0$ (market liquidity increases with providers' endowment) and $\frac{dM_f}{dw} < 0$ (mutual fund liquidity decreases with providers' endowment), confirming the strategic substitution pattern observed in Figure 5.

To derive the relationship between M_f and M_l directly, we can compute:

$$\frac{dM_f}{dM_l} = \frac{dM_f/dw}{dM_l/dw} = - \frac{\mathbb{E} \left[\frac{\partial^2 \rho^p}{\partial m_f \partial M_l} \right]}{\mathbb{E} \left[\frac{\partial^2 \rho^p}{\partial m_f^2} \right]} \cdot \frac{g''(w - M_l) + \mathbb{E} \left[\frac{\partial^2 v^p}{\partial m^2} \right]}{g''(w - M_l) - \mathbb{E} \left[\frac{\partial^2 v^p}{\partial m \partial M_f} \right] \cdot \frac{dM_f}{dw}} \quad (\text{B.36})$$

Since the concavity of the production functions implies $\frac{\partial^2 \rho^p}{\partial m_f^2} < 0$ and $\frac{\partial^2 v^p}{\partial m^2} < 0$, while complemen-

tarities between market and mutual fund liquidity suggest $\frac{\partial^2 \rho^p}{\partial m_f \partial M_l} > 0$ and $\frac{\partial^2 v^p}{\partial m \partial M_f} > 0$, we have $\frac{dM_f}{dM_l} < 0$, confirming the negative relationship observed in Figure 5.

B.1.2 Welfare analysis of endogenous mutual fund liquidity

To quantify the inefficiency in mutual funds' liquidity decisions, we formally derive the wedge between the marginal social and private values of liquidity across different equilibrium regions. This wedge is particularly pronounced in Region D, where mutual funds sell all their risky asset holdings but liquidity providers don't exhaust their liquid resources.

In this region, the marginal private value of mutual fund liquidity is:

$$\frac{\partial \rho^p}{\partial m_f} = f' \left(\frac{(1 - m_f)M_l}{1 - M_f} + m_f \right) \quad (\text{B.37})$$

The corresponding marginal social value internalizes the effect on equilibrium prices:

$$\frac{\partial \rho^s}{\partial M_f} = f' \left(\frac{(1 - M_f)M_l}{1 - M_f} + M_f \right) + \frac{\partial p^*}{\partial M_f} \cdot \frac{\partial \rho^s}{\partial p^*} \quad (\text{B.38})$$

Where $\frac{\partial p^*}{\partial M_f} > 0$ and $\frac{\partial \rho^s}{\partial p^*} > 0$, leading to $\frac{\partial \rho^s}{\partial M_f} - \frac{\partial \rho^p}{\partial m_f} > 0$.

B.2 Equilibrium with liquidity requirements

With a liquidity requirement m_f , a mutual fund's problem at date 1 is to choose $q \in [0, 1 - m_f]$ to maximize:

$$\max_{q \in [0, 1 - m_f]} \{f(qp + m_f) + (1 - m_f - q)\pi y\} \quad (\text{B.39})$$

The first-order condition for an interior solution is:

$$pf'(qp + m_f) = \pi y \quad (\text{B.40})$$

Solving for q yields the supply function:

$$q^S(p; \pi, m_f) = \frac{(f')^{-1}(\pi y/p) - m_f}{p} \quad (\text{B.41})$$

A corner solution with $q = 1 - m_f$ occurs when $p f'((1 - m_f)p + m_f) \geq \pi y$.

Given this supply function and liquidity providers' demand function (12), the equilibrium in the asset market can be characterized across different regions:

Region A: The equilibrium is at the intersection of the upward-sloping supply curve and the downward-sloping demand curve. Liquidity providers' binding resource constraint implies $p q = M_l$. Substituting into mutual funds' optimality condition:

$$p f'(M_l + M_f) = \pi y \quad (\text{B.42})$$

Solving for price and quantity:

$$p^*(\pi; M_l, M_f) = \frac{\pi y}{f'(M_l + M_f)} \quad \text{and} \quad q^*(\pi; M_l, M_f) = \frac{M_l f'(M_l + M_f)}{\pi y} \quad (\text{B.43})$$

This configuration obtains if $q^* < 1 - M_f$ and $p^* < \pi y$, which reduce to:

$$\frac{\pi y}{f'(M_l + M_f)} > \frac{M_l}{1 - M_f} \quad \text{and} \quad 1 < f'(M_l + M_f) \quad (\text{B.44})$$

Region B: The equilibrium is at the intersection of the upward-sloping supply curve and the flat part of the demand curve. The price equals the fundamental value:

$$p^*(\pi; M_l, M_f) = \pi y \quad \text{and} \quad q^*(\pi; M_l, M_f) = \frac{(f')^{-1}(1) - M_f}{\pi y} \quad (\text{B.45})$$

This obtains if $q^* < 1 - M_f$ and $p^* q^* < M_l$, which reduce to:

$$f'(\pi y(1 - M_f) + M_f) < 1 \quad \text{and} \quad (f')^{-1}(1) < M_l + M_f \quad (\text{B.46})$$

Region C: The equilibrium is at the intersection of the vertical supply curve and the flat part of the demand curve:

$$p^*(\pi; M_l, M_f) = \pi y \quad \text{and} \quad q^*(\pi; M_l, M_f) = 1 - M_f \quad (\text{B.47})$$

This obtains if $f'(\pi y(1 - M_f) + M_f) \geq 1$ and $p^* q^* < M_l$, which reduce to:

$$f'(\pi y(1 - M_f) + M_f) \geq 1 \quad \text{and} \quad \pi y(1 - M_f) < M_l \quad (\text{B.48})$$

Region D: The equilibrium is at the intersection of the vertical supply curve and the downward-sloping demand curve:

$$p^*(\pi; M_l, M_f) = \frac{M_l}{1 - M_f} \quad \text{and} \quad q^*(\pi; M_l, M_f) = 1 - M_f \quad (\text{B.49})$$

This obtains if $(1 - M_f)p^* f'((1 - M_f)p^* + M_f) \geq \pi y$ and $p^* < \pi y$, which reduce to:

$$M_l f'(M_l + M_f) \geq \pi y(1 - M_f) \quad \text{and} \quad M_l < \pi y(1 - M_f) \quad (\text{B.50})$$

To quantify the crowding-out effect of liquidity requirements, we differentiate liquidity providers' first-order condition with respect to M_f :

$$g''(w - M_l) \cdot \frac{dM_l}{dM_f} = - \frac{dv_0^{p'}(M_l + M_f)}{dM_f} \quad (\text{B.51})$$

This yields:

$$\frac{dM_l}{dM_f} = - \frac{v_0^{p'}(M_l + M_f)}{v_0^{p'}(M_l + M_f) + g''(w - M_l)} \quad (\text{B.52})$$

For interior values of M_f where mutual funds hold positive amounts of the risky asset and liquidity

providers have incentives to provide market liquidity, we have $v_0^{p'}(\cdot) > 0$ and $g''(\cdot) < 0$, which implies $-1 < \frac{dM_l}{dM_f} < 0$. This confirms that crowding-out is partial in the empirically relevant range where both mutual funds and liquidity providers are active market participants.

The effect on the ex-ante probability of fire sales follows from the equilibrium characterization within this interior range. The probability of fire sales is given by:

$$\Pr(\text{Fire Sale}) = \Pr(\pi > \pi^* \text{ and } M_l + M_f \leq \hat{M}) \quad (\text{B.53})$$

where π^* is the threshold value of π above which fire sales occur.

For moderate values of M_f where $\frac{d(M_l+M_f)}{dM_f} > 0$, the probability of fire sales decreases with the requirement, as shown in Figure 6(b) in the main text.

B.3 Welfare Analysis of liquidity requirements

Social welfare in the presence of liquidity requirements can be expressed as:

$$\Pi(M_l, M_f) = g(w - M_l) + M_l + \mathbb{E}[\pi y(1 - M_f) + f(I(\pi; M_l, M_f)) - I(\pi; M_l, M_f)] \quad (\text{B.54})$$

where $I(\pi; M_l, M_f) = p^*(\pi; M_l, M_f)q^*(\pi; M_l, M_f) + M_f$ denotes the total investment in the short-term project at date 1.

To quantify these effects, we calculate the derivative of social welfare with respect to the liquidity requirement M_f :

$$\begin{aligned} \frac{d\Pi}{dM_f} = & -\mathbb{E}[\pi y] + 1 + \frac{dM_l}{dM_f} [g'(w - M_l) - 1] \\ & + \mathbb{E} \left[\frac{\partial f(I(\pi; M_l, M_f))}{\partial M_f} - 1 + \frac{\partial f(I(\pi; M_l, M_f))}{\partial M_l} \frac{dM_l}{dM_f} - \frac{dM_l}{dM_f} \right] \end{aligned} \quad (\text{B.55})$$

The first term captures the direct portfolio effect, which is negative since $\mathbb{E}[\pi y] > 1$. The second

term reflects the direct benefit of having liquid assets available for investment in the short-term project. The third term captures the indirect effect through changes in liquidity providers' behavior, which is negative since $\frac{dM_l}{dM_f} < 0$ and $g'(w - M_l) > 1$ in equilibrium. The fourth term captures the resource allocation flexibility effect, which is positive in states where fire sales would otherwise occur.

Starting from the unregulated equilibrium ($M_f = 0$), a marginal increase in the liquidity requirement improves welfare if and only if:

$$\mathbb{E} \left[\frac{\partial f(I(\pi; M_l, 0))}{\partial M_f} - 1 \right] + \frac{dM_l}{dM_f} \left[\mathbb{E} \left[\frac{\partial f(I(\pi; M_l, 0))}{\partial M_l} - 1 \right] - (g'(w - M_l) - 1) \right] > \mathbb{E}[\pi y] - 1 \quad (\text{B.56})$$

This condition is derived by evaluating the derivative of welfare at $M_f = 0$ and requiring that it be positive. It shows that welfare improvements occur when the combined benefit of enhanced resource allocation flexibility (both direct and indirect) exceeds the opportunity cost of holding liquid assets instead of risky assets.

The optimal liquidity requirement M_f^* balances these effects, setting $\frac{d\Pi}{dM_f} = 0$. This yields:

$$\mathbb{E}[\pi y] - 1 = \mathbb{E} \left[\frac{\partial f(I(\pi; M_l, M_f))}{\partial M_f} - 1 \right] + \frac{dM_l}{dM_f} \left[\mathbb{E} \left[\frac{\partial f(I(\pi; M_l, M_f))}{\partial M_l} - 1 \right] - (g'(w - M_l) - 1) \right] \quad (\text{B.57})$$

For intermediate values of M_f , this equality can be satisfied, yielding an interior solution for the optimal liquidity requirement. However, as M_f increases further, the opportunity cost begins to dominate, resulting in a hump-shaped relationship between M_f and welfare, as shown in Figure 7 in the main text.

B.4 Derivation of implementation condition for liquidity requirements

For liquidity requirements to implement the constrained-efficient allocation, the resulting equilibrium must match the socially optimal levels:

$$M_l(M_f^*) = M_l^{SB} \quad \text{and} \quad M_f^* = M_f^{SB} \quad (\text{B.58})$$

The existence of such an equilibrium depends on the properties of the function $M_l(M_f)$, which describes how private liquidity provision responds to the regulatory requirement. From equation (B.52), we know:

$$\frac{dM_l}{dM_f} = -\frac{v_0^{p'}(M_l + M_f)}{v_0^{p'}(M_l + M_f) + g''(w - M_l)} \quad (\text{B.59})$$

Since $v_0^{p'}(\cdot) > 0$ and $g''(\cdot) < 0$, we have $-1 < \frac{dM_l}{dM_f} < 0$. This partial crowding-out effect implies that the function $T(M_f) = M_l(M_f) + M_f$ representing total liquidity is strictly increasing in M_f :

$$\frac{dT(M_f)}{dM_f} = \frac{dM_l}{dM_f} + 1 > 0 \quad (\text{B.60})$$

Therefore, for any total liquidity level between the unregulated level $M_l(0)$ and the maximum possible level (with $M_f = 1$), there exists a unique value of M_f that achieves it. However, matching the total liquidity level alone is insufficient for implementing the constrained-efficient allocation.

The constrained planner chooses M_f^{SB} and M_l^{SB} to maximize social welfare, with the first-order condition:

$$g'(w - M_l^{SB}) = v_0^{s'}(M^{SB}) \quad (\text{B.61})$$

In contrast, liquidity providers choose M_l based on their private incentives:

$$g'(w - M_l(M_f)) = v_0^{p'}(M_l(M_f) + M_f) \quad (\text{B.62})$$

For implementation, we need $M_l(M_f^{SB}) = M_l^{SB}$, which requires:

$$v_0^{p'}(M^{SB}) = v_0^{s'}(M^{SB}) \quad (\text{B.63})$$

However, due to the externality, these values generally differ. The liquidity requirement affects private incentives through the crowding-out effect. To find the condition under which implementation is possible, we differentiate the liquidity providers' optimality condition with respect to M_f :

$$-g''(w - M_l) \frac{dM_l}{dM_f} = v_0^{p''}(M_l + M_f) \left(\frac{dM_l}{dM_f} + 1 \right) \quad (\text{B.64})$$

At the constrained-efficient allocation, we can rearrange this to get:

$$\frac{dM_l}{dM_f} = \frac{-v_0^{p''}(M^{SB})}{g''(w - M_l^{SB}) + v_0^{p''}(M^{SB})} \quad (\text{B.65})$$

For implementation to be possible, this must equal the ratio of the wedge between private and social values to the private value:

$$\left. \frac{dM_l}{dM_f} \right|_{(M_f^{SB}, M_l^{SB})} = \frac{v_0^{p'}(M^{SB}) - v_0^{s'}(M^{SB})}{v_0^{p'}(M^{SB})} \quad (\text{B.66})$$

This condition has a precise economic interpretation, as stated in part (iii) of Proposition 8: the requirement can implement the constrained-efficient allocation if and only if the proportional reduction in private liquidity (due to crowding-out) exactly offsets the proportional wedge between private and social values of liquidity.

B.5 Equilibrium with redemption gates

Redemption gates constrain the price of the risky asset to be no less than $(1 - \delta)\pi y$, where $\delta \in [0, 1]$ is the gate threshold. The gates modify the equilibrium particularly in Regions A and D, where fire sales occur.

Region A (modified): In the unregulated equilibrium, this region features $p^* = \pi y / f'(M)$ and $q^* = M f'(M) / \pi y$. With gates, the price floor becomes binding if $p^* < (1 - \delta)\pi y$, which occurs when $f'(M) > 1 / (1 - \delta)$. If the constraint binds, the equilibrium becomes:

$$p^* = (1 - \delta)\pi y \quad \text{and} \quad q^* = \frac{M_l}{(1 - \delta)\pi y} \quad (\text{B.67})$$

This equilibrium obtains if $q^* < 1$, which reduces to $M_l < (1 - \delta)\pi y$.

Region D (modified): In the unregulated equilibrium, this region features $p^* = M$ and $q^* = 1$. With gates, the price floor becomes binding if $M < (1 - \delta)\pi y$. If the constraint binds, the equilibrium becomes:

$$p^* = (1 - \delta)\pi y \quad \text{and} \quad q^* = \frac{M_l}{(1 - \delta)\pi y} \quad (\text{B.68})$$

If $M_l / (1 - \delta)\pi y \geq 1$, then mutual funds sell all their risky assets: $q^* = 1$. This occurs when $M_l \geq (1 - \delta)\pi y$. For liquidity providers, gates alter the marginal private value of liquidity by capping fire-sale discounts. When gates bind, the marginal private value becomes:

$$\frac{\partial v^P(\pi; m, M)}{\partial m} = \frac{\delta}{1 - \delta} \quad (\text{B.69})$$

This fixed premium is determined solely by the gate threshold δ . As δ decreases, the potential return from holding liquidity diminishes, reducing the ex-ante incentives for liquidity provision. This explains the strong negative relationship between the gate threshold and equilibrium market liquidity, as stated in part (ii) of Proposition 9: $\frac{dM_l}{d\delta} > 0$.

To see why this relationship holds, note that liquidity providers' first-order condition can be written as:

$$g'(w - M_l) = 1 + \mathbb{E} \left[\left(\frac{\partial v^P(\pi; m, M)}{\partial m} - 1 \right) \cdot \mathbb{1}_{gate} \right] \quad (\text{B.70})$$

where $\mathbb{1}_{gate}$ is an indicator for states where gates bind. As δ decreases, the marginal private value $\frac{\delta}{1 - \delta}$ decreases, reducing the expected return from holding liquidity. This shifts the right-hand side

of the first-order condition downward, leading to a lower equilibrium value of M_l .

B.6 Welfare Analysis of Redemption Gates

This section provides a formal welfare analysis of redemption gates, supporting Proposition 10 in the main text.

Social welfare with redemption gates can be written as:

$$\Pi(M_l, \delta) = g(w - M_l) + M_l + \mathbb{E}[\pi y + f(I(\pi; M_l, \delta)) - I(\pi; M_l, \delta)] \quad (\text{B.71})$$

where $I(\pi; M_l, \delta) = \min\{p^*(\pi; M_l, \delta)q^*(\pi; M_l, \delta), M_l\}$ represents the investment in the short-term project.

To analyze the welfare effects of redemption gates, we examine the derivative of welfare with respect to the gate threshold δ :

$$\frac{d\Pi}{d\delta} = \frac{dM_l}{d\delta} \left[g'(w - M_l) - 1 + \mathbb{E} \left[\frac{\partial f(I)}{\partial M_l} - 1 \right] \right] + \mathbb{E} \left[\frac{\partial f(I)}{\partial \delta} - \frac{\partial I}{\partial \delta} \right] \quad (\text{B.72})$$

This expression decomposes the welfare effect into two components:

The first term captures the indirect effect of gates on welfare through changes in market liquidity. Since $\frac{dM_l}{d\delta} > 0$ (liquidity decreases as gates become tighter, that is as δ decreases) and $g'(w - M_l) > 1$ in equilibrium, this term is positive—reduced liquidity provision lowers welfare.

We can examine each equilibrium region to understand when this direct effect is positive:

- In regions where $p^* \geq (1 - \delta)\pi y$ (gates do not bind), we have $\frac{\partial I}{\partial \delta} = 0$, so there is no direct effect.
- In Region A where gates bind and mutual funds sell some but not all of their risky assets, the gates improve welfare when $f'(I) > \frac{1}{1 - \delta}$.

- In Region D where gates bind and mutual funds sell all their risky assets, the welfare effect depends on the magnitudes of the fire-sale discount and the marginal product of the short-term investment.

Of particular interest is the welfare effect of a marginal introduction of gates, evaluated at $\delta = 1$ (the unregulated case):

$$\left. \frac{d\Pi}{d\delta} \right|_{\delta=1} = \frac{dM_l}{d\delta} \left[g'(w - M_l) - 1 + \mathbb{E} \left[\frac{\partial f(I)}{\partial M_l} - 1 \right] \right] + \mathbb{E} \left[\frac{\partial f(I)}{\partial \delta} - \frac{\partial I}{\partial \delta} \right] \quad (\text{B.73})$$

Gates improve welfare if $\left. \frac{d\Pi}{d\delta} \right|_{\delta=1} < 0$, which requires:

$$\mathbb{E} \left[\frac{\partial f(I)}{\partial \delta} - \frac{\partial I}{\partial \delta} \right] < -\frac{dM_l}{d\delta} \left[g'(w - M_l) - 1 + \mathbb{E} \left[\frac{\partial f(I)}{\partial M_l} - 1 \right] \right] \quad (\text{B.74})$$

This condition aligns with parts (i) and (ii) of Proposition 10, which decompose the welfare effect into indirect and direct components. The indirect effect through reduced market liquidity is generally negative, while the direct effect can be positive in states with extreme fire sales.

B.7 Comparing regulatory interventions

For gates to implement the constrained-efficient allocation, there must exist a threshold δ^* such that:

$$M_l(\delta^*) = M_l^{SB} \quad \text{and} \quad I(\pi; M_l(\delta^*), \delta^*) = I^{SB}(\pi) \quad \forall \pi \quad (\text{B.75})$$

where $M_l(\delta)$ is liquidity providers' optimal response function, $I(\pi; M_l, \delta)$ is the resulting investment in the short-term project, and $I^{SB}(\pi)$ is the constrained-efficient investment function.

Gates fundamentally alter the state-contingent allocation of resources. The investment in the

short-term project under gates is:

$$I(\pi; M_l, \delta) = \begin{cases} \min\{p^*(\pi; M_l)q^*(\pi; M_l), M_l\} & \text{if } p^*(\pi; M_l) \geq (1 - \delta)\pi y \\ (1 - \delta)\pi y \cdot q^{Gate}(\pi; \delta) & \text{if } p^*(\pi; M_l) < (1 - \delta)\pi y \end{cases} \quad (\text{B.76})$$

where $q^{Gate}(\pi; \delta) \leq 1$ is the constrained quantity of assets that mutual funds can sell when gates bind.

For gates to implement the constrained-efficient allocation, the threshold δ^* would need to simultaneously achieve:

$$(1 - \delta^*)\pi y \cdot q^{Gate}(\pi; \delta^*) = I^{SB}(\pi) \quad \forall \pi \text{ where gates bind} \quad (\text{B.77})$$

This presents a fundamental limitation, as stated in part (iii) of Proposition 11: the constrained-efficient investment function $I^{SB}(\pi)$ varies continuously with π , while gates impose a discrete constraint on resource reallocation that depends only on the gate threshold δ . Consequently, equation (B.77) cannot generally be satisfied for all relevant states with a single gate threshold.